ECS 455: Quiz 4 Solution

Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID
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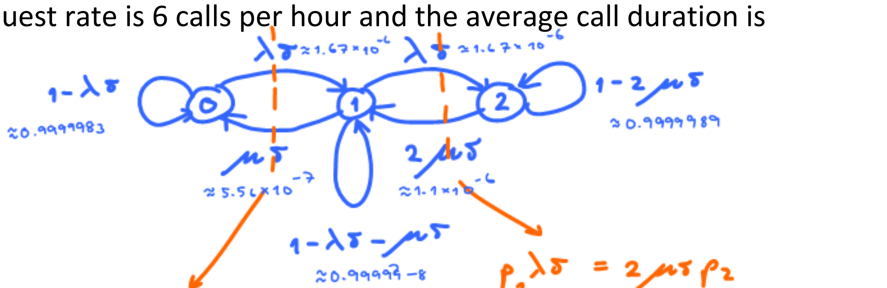
Consider a system which has 2 channels. We would like to find the blocking probability via the Markov chain method. For each of the following models, **<u>draw the Markov chain</u>** via discrete time approximation. Don't forget to indicate the transition probabilities on the arrows. Assume that the duration of each time slot is 1 millisecond. Then, use global balance equations to find (1) the **<u>steady-state probabilities</u>** and then (2) the long-term **<u>call blocking probability</u>**.

$\delta = 10^{-3}$ sec

1. ***Erlang B*** model: Assume that the *<u>total</u>* call request rate is 6 calls per hour and the average call duration is 30 mins.



$\lambda = 6 \frac{\text{calls}}{\text{hr}} = \frac{6}{60} \frac{\text{calls}}{\text{min}} = \frac{6}{3600} \frac{\text{calls}}{\text{sec}}$

$\lambda\delta = \frac{6}{3600} \times 10^{-3} = \frac{1}{6} \times 10^{-5} \approx 1.67\times10^{-6}$

$\mu\delta = \frac{1}{30\times60} \times 10^{-3} = \frac{1}{18} \times 10^{-5} \approx 5.56\times10^{-7}$

$A = \frac{\lambda}{\mu} = \lambda \times \frac{1}{\mu} = \frac{6}{60} \times 30 = 3$ Erlangs

$p_0 \lambda\delta = p_1 \mu\delta$

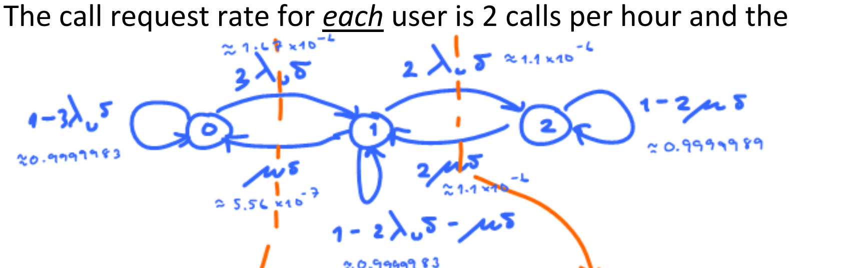
$p_1 = A p_0$

$p_1 \lambda\delta = 2\mu\delta p_2$

$p_2 = \frac{A}{2} p_1 = \frac{A^2}{2} p_0$

$p_0 + p_1 + p_2 = 0 \Rightarrow p_0 = \frac{1}{1+A+\frac{A^2}{2}} = \frac{1}{1+3+\frac{3^2}{2}} = \frac{2}{17}$ (≈ 0.118) $\Rightarrow p_1 = 3 \times \frac{2}{17} = \frac{6}{17}$ (≈ 0.353), $p_b = p_2 = \frac{3}{2} \times \frac{6}{17} = \frac{9}{17}$ (≈ 0.529).

2. ***Engset*** model: Assume that there are 3 users. The call request rate for *<u>each</u>* user is 2 calls per hour and the average call duration is 30 mins.



$\lambda_u = 2 \frac{\text{calls}}{\text{hr}}$ ← observe that this is $\frac{\lambda}{3}$

$\lambda_u\delta = \frac{\lambda}{3}\delta = \frac{1}{18} \times 10^{-5} \approx 5.56\times10^{-7}$

$\mu\delta = \frac{1}{18} \times 10^{-5} \approx 5.56\times10^{-7}$ ← same as in Q1.

$A_u = \frac{\lambda_u}{\mu} = \frac{\lambda}{3\mu} = \frac{A}{3} = \frac{3}{3} = 1$ Erlang.

$p_0 3\lambda_u\delta = p_1 \mu\delta$

$p_1 = 3A_u p_0$

$p_1 2\lambda_u\delta = p_2 2\mu\delta$

$p_2 = A_u p_1 = 3A_u^2 p_0$

$p_0 + p_1 + p_2 = 1 \Rightarrow p_0 = \frac{1}{1+3A_u+3A_u^2} = \frac{1}{7} \approx 0.143$

$p_1 = 3A_u p_0 = 3p_0 = \frac{3}{7} \approx 0.429$

$p_2 = 3A_u^2 p_0 = 3p_0 = \frac{3}{7} \approx 0.429$

$p_b = \frac{\frac{3}{7} \times \lambda_u\delta}{\frac{1}{7} \times 3\lambda_u\delta + \frac{3}{7} \times 2\lambda_u\delta + \frac{3}{7} \times \lambda_u\delta} = \frac{3}{3+6+3} = \frac{1}{4} \approx 0.25$