ECS 455: Quiz 3 Solution

Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID
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$m=2$

Consider a system which has 2 channels. We would like to find the blocking probability via the Markov chain method. For each of the following models, **draw the Markov chain** via discrete time approximation. Don't forget to indicate the transition probabilities on the arrows. Assume that the duration of each time slot is 1 millisecond. Then, use global balance equations to find (1) the **steady-state probabilities** and then (2) the long-term **call blocking probability**.

$\delta = 10^{-3}$

$\lambda = 12$ calls/hour

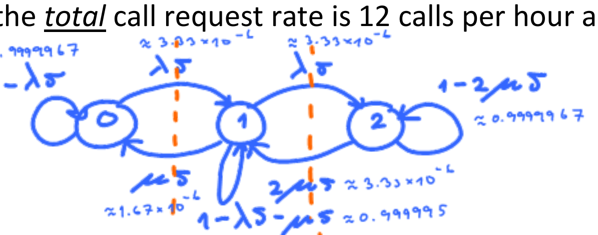
1. ***Erlang B*** model: Assume that the *total* call request rate is 12 calls per hour and the average call duration is 10 mins.

$\frac{1}{\mu} =$ 10 mins

$\lambda\delta = \frac{\cancel{12}^{2} \times 10^{-3}}{\cancel{60} \times 60} = \frac{10^{-5}}{3} \approx 3.33\times10^{-6}$

$\mu\delta = \frac{1}{10\text{ min}} \times 10^{-3}\text{ sec} = \frac{10^{-3}}{10\times 60} = \frac{10^{-5}}{6} \approx 1.67\times10^{-6}$

$A = \frac{\lambda}{\mu} = \frac{\cancel{12}^{2}}{\cancel{60}} \times 1\cancel{0} = 2$ Erlangs



$\lambda\cancel{\delta} p_0 = \mu\cancel{\delta} p_1$

$p_1 = \frac{\lambda}{\mu} p_0 = A p_0$

$\lambda\cancel{\delta} p_1 = 2\mu\cancel{\delta} p_2$

$p_2 = \frac{1}{2}\frac{\lambda}{\mu} p_1 = \frac{1}{2} A p_1 = \frac{1}{2} A^2 p_0$

$p_0 + p_1 + p_2 = 1$

$p_0 + A p_0 + \frac{1}{2} A^2 p_0 = 1$

$p_0 = \frac{1}{1 + A + \frac{1}{2}A^2} = \boxed{\frac{1}{5}} \approx 0.2$

$p_1 = \boxed{\frac{2}{5}} \approx 0.4$

$p_2 = \frac{1}{\cancel{2}} \times 2^{\cancel{2}} \times \frac{1}{5} = \boxed{\frac{2}{5}} \approx 0.4$

$p_b = p_m = p_2 = \boxed{\frac{2}{5}} \approx 0.4$

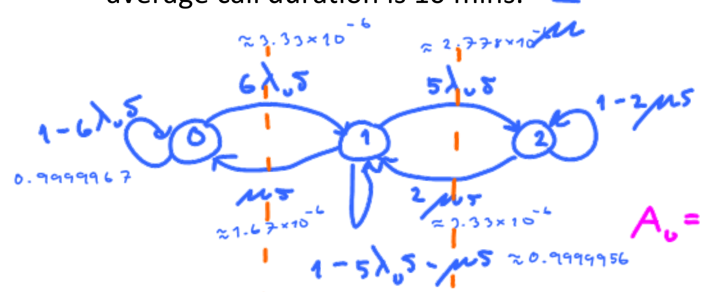
2. ***Engset*** model: Assume that there are 6 users. The call request rate for *each* user is 2 calls per hour and the average call duration is 10 mins.

$n=6$, $\lambda_u = 2$, $= \frac{1}{\mu}$

$\lambda_u\delta = \frac{2}{(60)^2} \times 10^{-3} = \frac{1}{18} \times 10^{-5}$

$\mu\delta = \frac{10^{-5}}{6}$

$A_u = \frac{\lambda_u}{\mu} = \frac{2}{60} \times 10 = \frac{1}{3}$



Global balance equation

$6\lambda_u\cancel{\delta} p_0 = \mu\cancel{\delta} p_1$

$p_1 = 6 A_u p_0$

$5\lambda_u\cancel{\delta} p_1 = 2\mu\cancel{\delta} p_2$

$p_2 = \frac{5}{2} A_u p_1 = 15 A_u^2 p_0$

$p_0 + p_1 + p_2 = 1$

$p_0 + 6A_u p_0 + 15 A_u^2 p_0 = 1$

$p_0 = \frac{1}{1 + 6A_u + 15A_u^2} = \frac{1}{1 + 6\cdot\frac{1}{3} + \cancel{15}^{5} \times \frac{1}{\cancel{9}_{3}}} = \frac{14}{3} = \boxed{\frac{3}{14}} \approx 0.214$

$p_1 = 6 \cdot \frac{1}{3} \cdot \frac{3}{14} = \boxed{\frac{3}{7}} \approx 0.429$

$p_2 = \cancel{15}^{5} \times \frac{1}{\cancel{9}} \times \frac{\cancel{3}}{14} = \boxed{\frac{5}{14}} \approx 0.357$

$p_b = \frac{4\cancel{\lambda_u\delta} p_2}{6\cancel{\lambda_u\delta} p_0 + 5\cancel{\lambda_u\delta} p_1 + 4\cancel{\lambda_u\delta} p_2} = \frac{4\cdot\frac{5}{14}}{6\cdot\frac{3}{14} + 5\cdot\frac{6}{14} + 4\cdot\frac{5}{14}}$

$= \frac{20}{18 + 30 + 20} = \frac{20}{68} = \boxed{\frac{5}{17}} \approx 0.294$