3.3 Markov Chain and Erlang B formula

Monday, January 07, 2013 10:43 AM

① Steady-state probabilities → p_i

Q: If I look at the system at some random time, what is the probability that the system will be in state $K=i$?

A: p_i

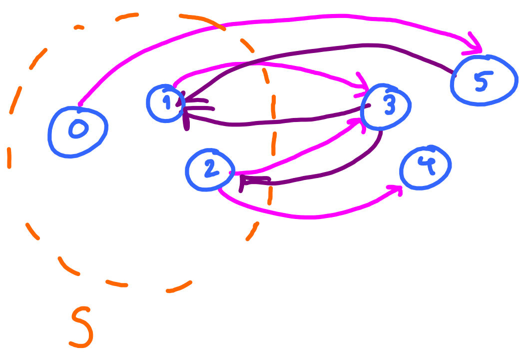
To find p_i's, we have two techniques:

a) Let the system evolve. Find the limits.

b) Use balance equations → solve equations.

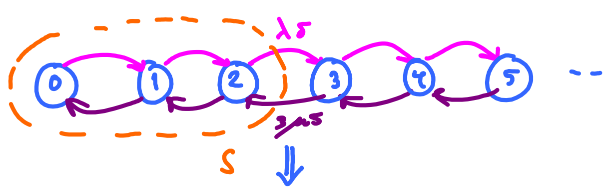
The version of balance equation that we used in class relies on "global" balance.

Each global balance equation comes from selecting a collection S of state and then balance the probability flux in and out of this collection.



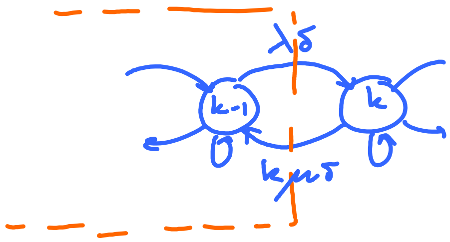
probability "flux" that goes into S = "flux" that goes out of S

For the Erlang-B system,



⇓

$$p_2 \lambda \delta = p_3 3\mu\delta$$



$$\Rightarrow p_{k-1}\,\lambda\delta = p_k\,k\mu\delta$$

$$p_k = \frac{\lambda}{\mu k}\,p_{k-1} = \frac{A}{k}\,p_{k-1}$$

$$p_1 = \frac{A}{1}\,p_0$$

$$p_2 = \frac{A}{2}\,p_1 = \frac{A}{2}\,\frac{A}{1}\,p_0$$

$$p_3 = \frac{A}{3}\,p_2 = \frac{A}{3}\,\frac{A}{2}\,\frac{A}{1}\,p_0$$

$$\vdots$$

$$p_k = \frac{A^k}{k!}\,p_0$$

use the fact that the sum of p_k should be 1.

$$1 = \sum_{k=0}^{m} p_k = p_0 \sum_{k=0}^{m} \frac{A^k}{k!} \Rightarrow p_0 = \frac{1}{\sum_{k=0}^{m} \frac{A^k}{k!}} \Rightarrow p_k = \frac{A^k/k!}{\sum_{i=0}^{m} \frac{A^i}{i!}}$$