3.3 Steady-State Probabilities for Markov Chain

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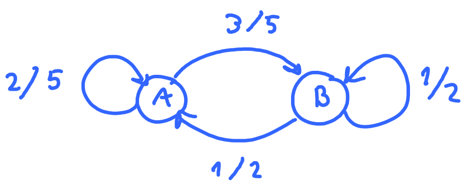
Analyzing a simple Markov chain

Consider a system governed by the Markov chain below



Assume that at time $t=0$, the system is in state B.

Let the system evolves according to the transition probabilities in the Markov chain.

Exp. #	time [δ] →	0	1	2	3	4	5	...		
1		B	A	A	B	B	B	A	B	A ...
2		B	A	A	A	B	A	B	B	B ...
⋮		B								
⋮		⋮								
100,000		B								

Look at time t far away from $t=0$.
Among the 100,000 experiments, how many of them are in state A?
how many are in state B?

Probabilistically, we talk about

relative freq.

$$f_A = \frac{\# A}{10^5} \xrightarrow{LLN} p_A$$

$$f_B = \frac{\# B}{10^5} \xrightarrow{LLN} p_B$$

(the convergence from relative freq. to probability values comes from LLN)

	A	B
$t=0$	0	100,000
	$\times 1/2$	$\times \frac{1}{2}$
$t=\delta$	50,000	50,000

$t = \delta$ $\qquad$ 50,000 $\qquad$ 50,000

$\times \frac{2}{5}$ $\qquad$ 1/2 $\qquad$ 1/2

$t = 2\delta$ $\qquad$ 20,000 + 25,000 = 45,000 $\qquad$ 25,000 + 30,000 = 55,000

$t = 3\delta$ $\qquad$ 45,500 $\qquad$ 54,500

$t = 4\delta$ $\qquad$ 45450 $\qquad$ 54,550

$t = 5\delta$ $\qquad$ 45455 $\qquad$ 54545

45454.5 $\qquad$ 54545.5

$t = t_{large}$ (some "large" value) $\qquad$ n_A $\qquad$ n_B

3/5 $\qquad$ 1/2

$t = t_{large} + \delta$ $\qquad$ n_A $\qquad$ n_B

If the system somehow reaches equilibrium, then we must have

$$\frac{3}{5} n_A = \frac{1}{2} n_B \Rightarrow n_B = \frac{6}{5} n_A$$

$$n_A + n_B = 10^5$$

$$n_A + \frac{6}{5} n_A = 10^5$$

$$n_A = 10^5 \times \frac{5}{11}$$

$$n_B = 10^5 - 10^5 \times \frac{5}{11} = 10^5 \times \frac{6}{11}$$

Another view:

Look directly at the probability.

$$\frac{3}{5} p_A = \frac{1}{2} p_B \Rightarrow p_B = \frac{6}{5} p_A$$

$$p_A + p_B = 1$$

$$p_A + \frac{6}{5} p_A = 1$$

$$\frac{11}{5} p_A = 1$$

$$p_A = \frac{5}{11}, \quad p_B = \frac{6}{11}$$

Note: The values of n_A and n_B are actually random.

The values we found above are simply their approximation

As we increase the ~~#~~ experiments

As we increase the # experiments, then $f_A = \frac{n_A}{\# \text{ experiments}}$ and f_B will converge to p_A and p_B respectively by the law of large numbers.