

# ECS455 Chapter 2

## Cellular Systems

### 2.2 Co-Channel Interference

Dr. Prapun Suksompong  
[prapun.com/ecs455](http://prapun.com/ecs455)

**Office Hours:**

**BKD 3601-7**

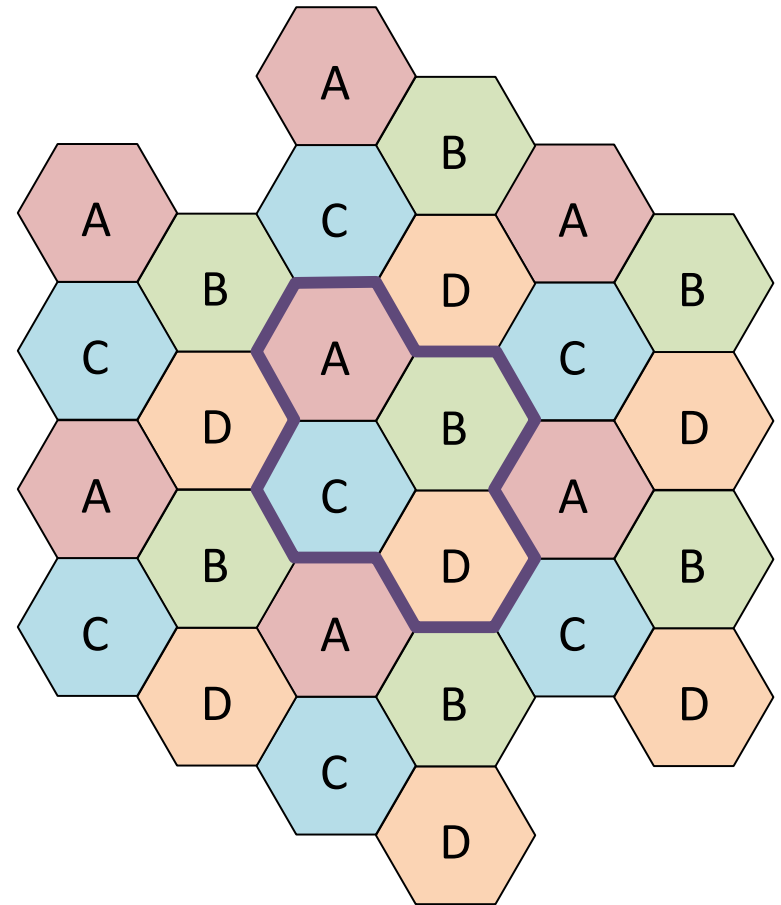
**Monday 9:20-10:20**

**Wednesday 9:20-10:20**

(Inter-cell)

# Co-Channel Interference

- Frequency reuse  $\rightarrow$  co-channel interference
- Consider only nearby interferers.
  - Power decreases rapidly as the distance increases.
- In a **fully equipped hexagonal-shaped** cellular system, there are always  $K = 6$  cochannel interfering cells in the **first tier**.



# Three Measures of Signal Quality

- Old (**noise-limited** systems)  $\text{SNR} = \frac{P_r}{P_{\text{noise}}}$
- Consider both noise & interference  $\text{SINR} = \frac{P_r}{P_{\text{interference}} + P_{\text{noise}}}$
- The best cellular system design places users that share the same channel at a separation distance (as close as possible) Why? where the intercell interference is just below the maximum tolerable level for the required data rate and BER.
- Good cellular system designs are interference-limited, meaning that the interference power is much larger than the noise power.

$$\text{SIR} = \frac{P_r}{P_{\text{interference}}}$$

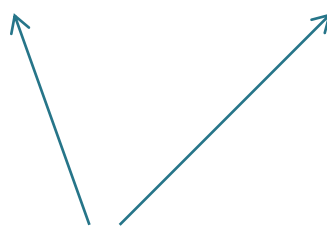
# “Reliable”/“tolerable”?

(Why not as far as possible?)

Co-channel cells, must be spaced **far enough** apart so that interference between users in co-channel cells does not degrade **signal quality** below tolerable levels.

Subjective tests found that people regard an FM signal using a 30 kHz channel bandwidth to be clear if the signal power is at least **sixty times** higher than the noise/interference power.

[Klemens, 2010, p 54]

$$10\log_{10} 60 = 17.78 \approx 18 \text{ dB}$$


We will soon revisit and use these numbers for some more specific calculations

# Review: Simplified Path Loss Model

$$\frac{P_r}{P_t} = K \left( \frac{d_0}{d} \right)^\gamma \quad \longrightarrow \quad P_r = \frac{P_t K d_0^\gamma}{d^\gamma} \propto \frac{1}{d^\gamma}$$

Captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

- $K$  is a unitless constant which depends on the antenna characteristics and the average channel attenuation
- $d_0$  is a reference distance for the antenna far-field
  - Typically 1-10 m indoors and 10-100 m outdoors.
- $\gamma$  is the **path loss exponent**.
  - 2 in free-space model
  - 4 in two-ray model [Goldsmith, 2005, eq. 2.17]

Environment	$\gamma$ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

[Goldsmith, 2005, Table 2.2]

# SIR (S/I): Definition/Calculation

- $K = \#$  co-channel interfering cells
- The **signal-to-interference ratio** (S/I or SIR) for a mobile receiver which monitors a forward channel can be expressed as

$$\text{SIR} = \frac{P_r \leftarrow \text{Signal}}{P_{\text{interference}}} = \frac{P_r}{\sum_{i=1}^K P_{\text{of the } i^{\text{th}} \text{ interferer}}}$$

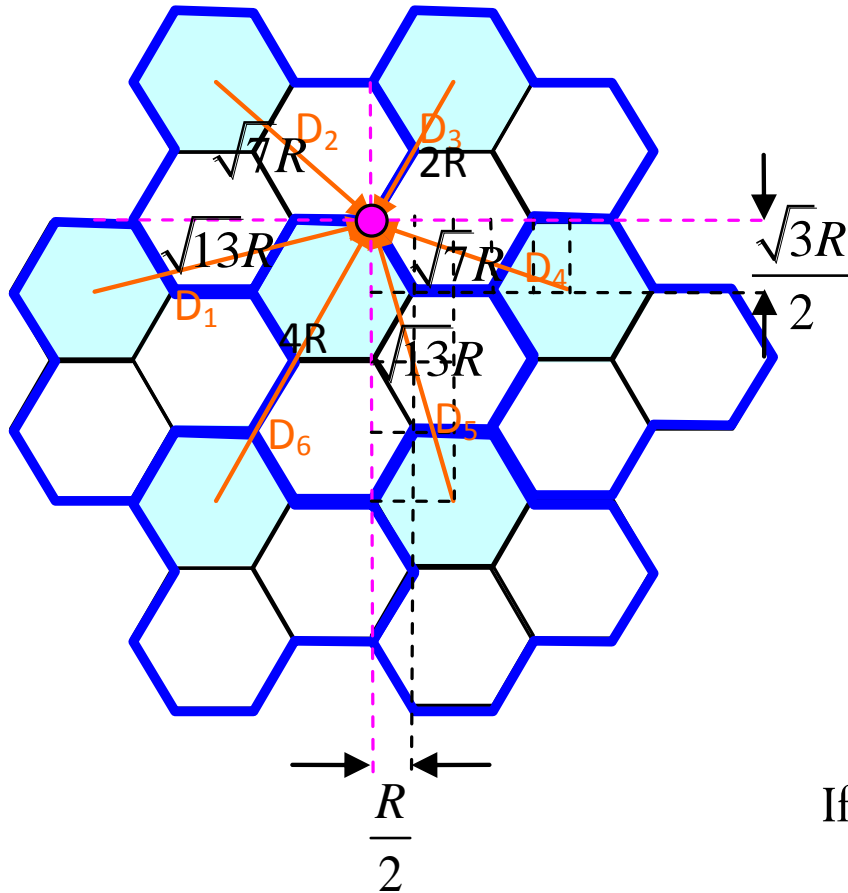
- $P_r$  = the desired signal **power** from the desired base station
- $P_i$  = the interference **power** caused by the  $i$ th interfering co-channel cell base station.
- Often called the **carrier-to-interference ratio**: CIR.

# SIR Threshold

- The SIR should be greater than a specified threshold for proper signal operation.
- In the 1G **AMPS** system, designed for **voice** calls, the threshold for acceptable voice quality is SIR equal to **18 dB**.
- For the 2G digital AMPS system (D-AMPS or IS-54/136), a threshold of 14 dB is deemed suitable.
- For the **GSM** system, a range of **7–12 dB**, depending on the study done, is suggested as the appropriate threshold.
- The probability of error in a digital system depends on the choice of this threshold as well.

# SIR: N = 3

(Ignore co-channel cells that are too far away)



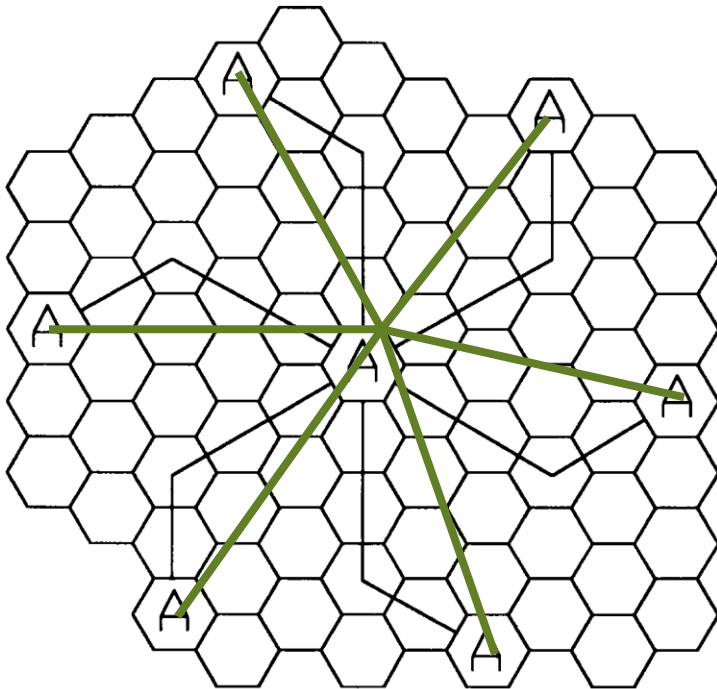
- Consider only first tier.
- Worse-case distance

$$\begin{aligned}
 \text{SIR} &\approx \frac{k/R^\gamma}{\sum_i k/D_i^\gamma} = \frac{1}{\sum_i 1/\left(\frac{D_i}{R}\right)^\gamma} = \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}} \\
 &= \frac{1}{2(\sqrt{7})^{-\gamma} + 2(\sqrt{13})^{-\gamma} + 2^{-\gamma} + 4^{-\gamma}}
 \end{aligned}$$

If N = 19, will the SIR be better or worse?



# Approximation

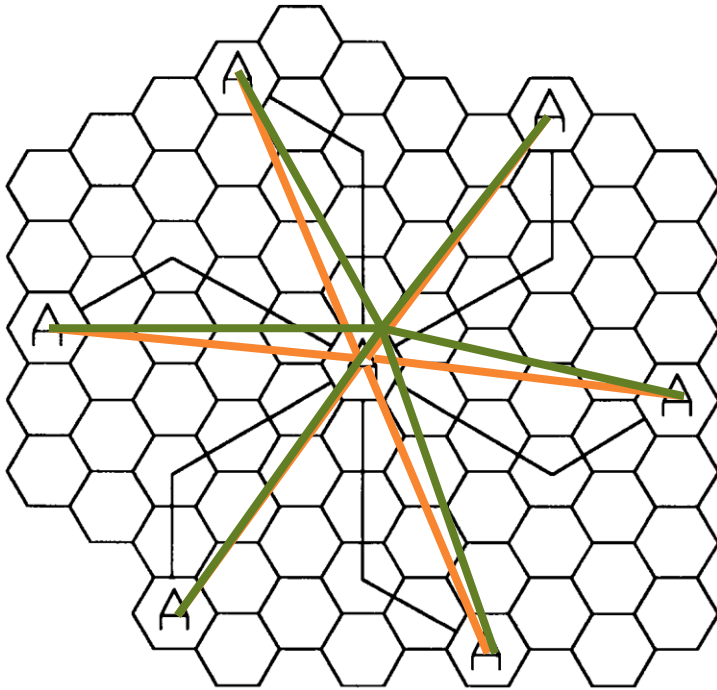


- Consider only first tier.
- Worse-case distance

$$\text{SIR} \approx \frac{1}{\sum_i \left( \frac{D_i}{R} \right)^{-\gamma}}$$

- Use the same  $D$  for  $D_i$

# Approximation



- Consider only first tier.
- Worse-case distance

$$\text{SIR} \approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}}$$

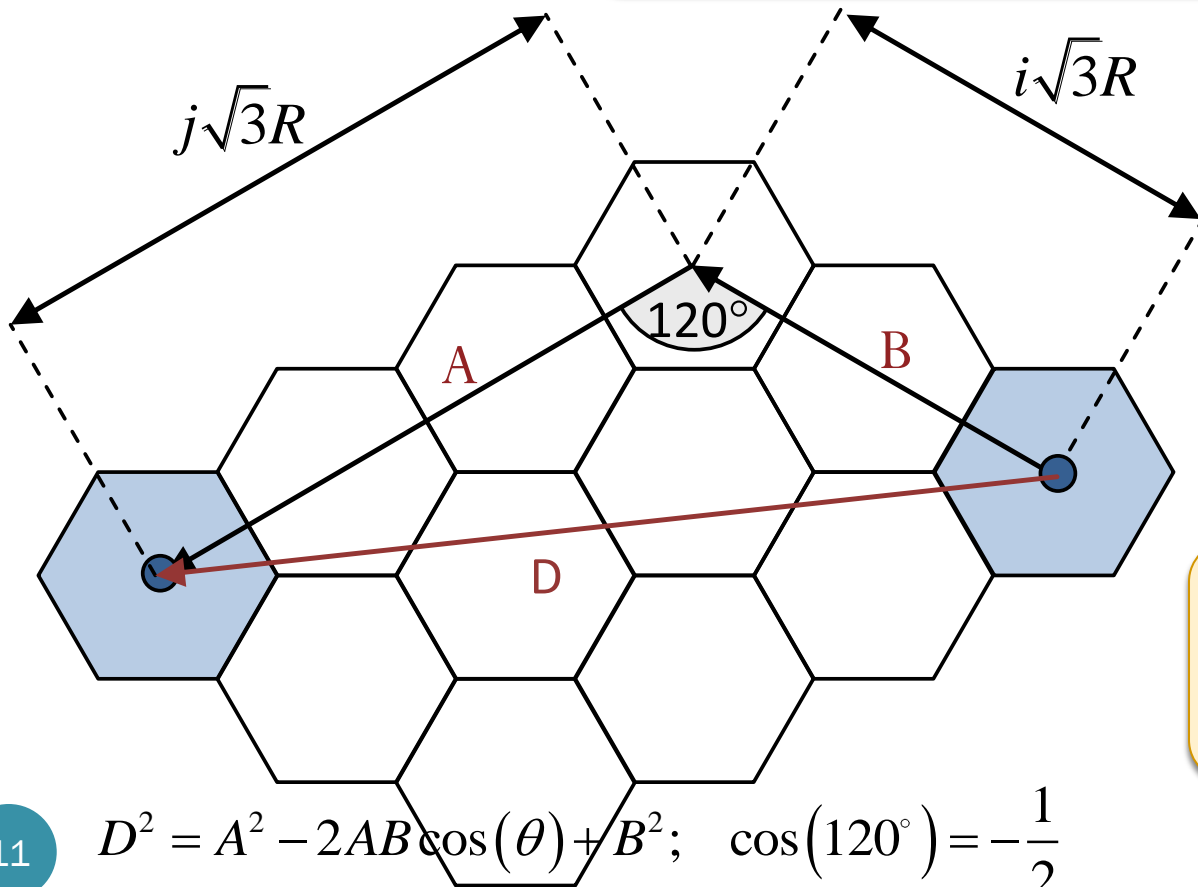
- Use the same  $D$  for  $D_i$

$$\text{SIR} \approx \frac{1}{\sum_i \left(\frac{D}{R}\right)^{-\gamma}} \approx \frac{1}{K \left(\frac{D}{R}\right)^{-\gamma}} = \frac{1}{K} \left(\frac{D}{R}\right)^{\gamma}$$

Notice that  $D/R$  is an important quantity!

# Center-to-center distance (D)

$$D = \sqrt{(i\sqrt{3}R)^2 + (j\sqrt{3}R)^2 - 2(i\sqrt{3}R)(j\sqrt{3}R)\cos(120^\circ)}$$
$$= R\sqrt{3(i^2 + j^2 + ij)} = R\sqrt{3N}$$



This distance,  $D$ , is called **reuse distance**.

Co-channel reuse ratio

$$Q = \frac{D}{R} = \sqrt{3N}.$$

$$D^2 = A^2 - 2AB\cos(\theta) + B^2; \quad \cos(120^\circ) = -\frac{1}{2}$$

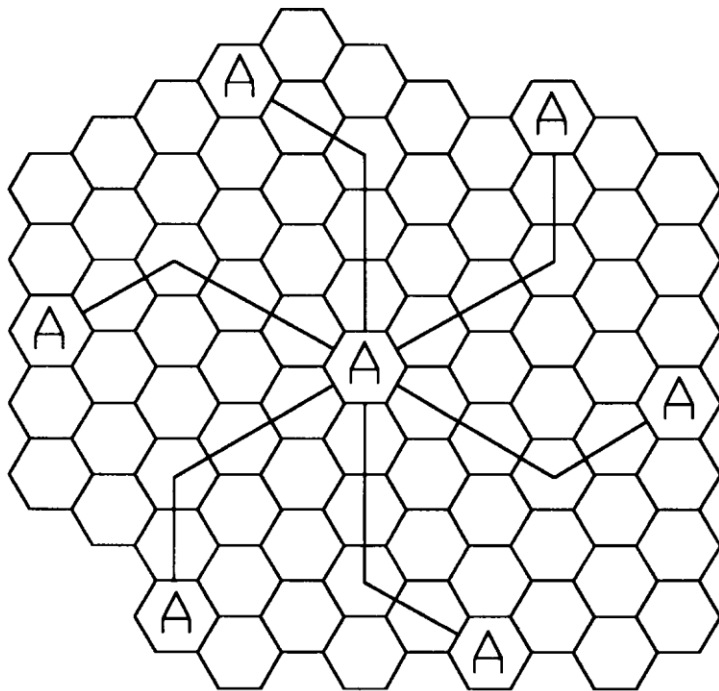
# Q and N

**Co-channel reuse ratio**

$$Q = \frac{D}{R} = \sqrt{3N}.$$

	<b>Cluster Size (<math>N</math>)</b>	<b>Co-channel Reuse Ratio (<math>Q</math>)</b>
$i = 1, j = 1$	3	3
$i = 1, j = 2$	7	4.58
$i = 0, j = 3$	9	5.20
$i = 2, j = 2$	12	6

# Approximation: Crude formula

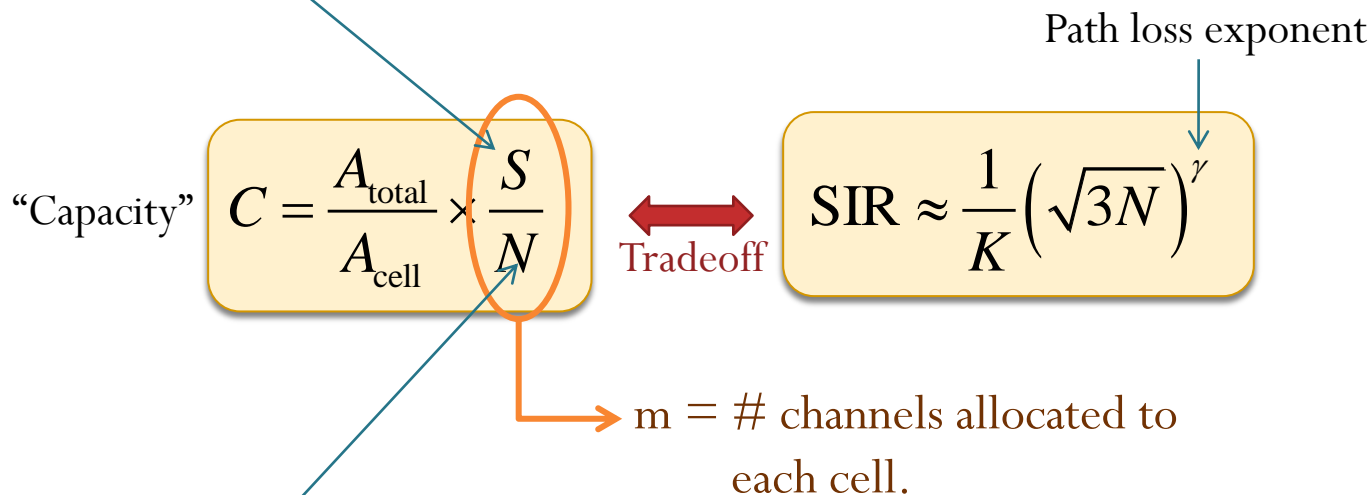


$$\begin{aligned} \text{SIR} &= \frac{P_r}{P_{\text{interference}}} = \frac{P_r}{\sum_{i=1}^K P_{\text{of the } i^{\text{th}} \text{ interferer}}} \\ &\approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}} \approx \frac{1}{K \left(\frac{D}{R}\right)^{-\gamma}} = \frac{1}{K} \left(\frac{D}{R}\right)^{\gamma} \\ &= \frac{1}{K} \left(\sqrt{3N}\right)^{\gamma} \end{aligned}$$

As the cell cluster size ( $N$ ) increases, the spacing ( $D$ ) between interfering cells increases, reducing the interference.

# Summary: Quantity vs Quality

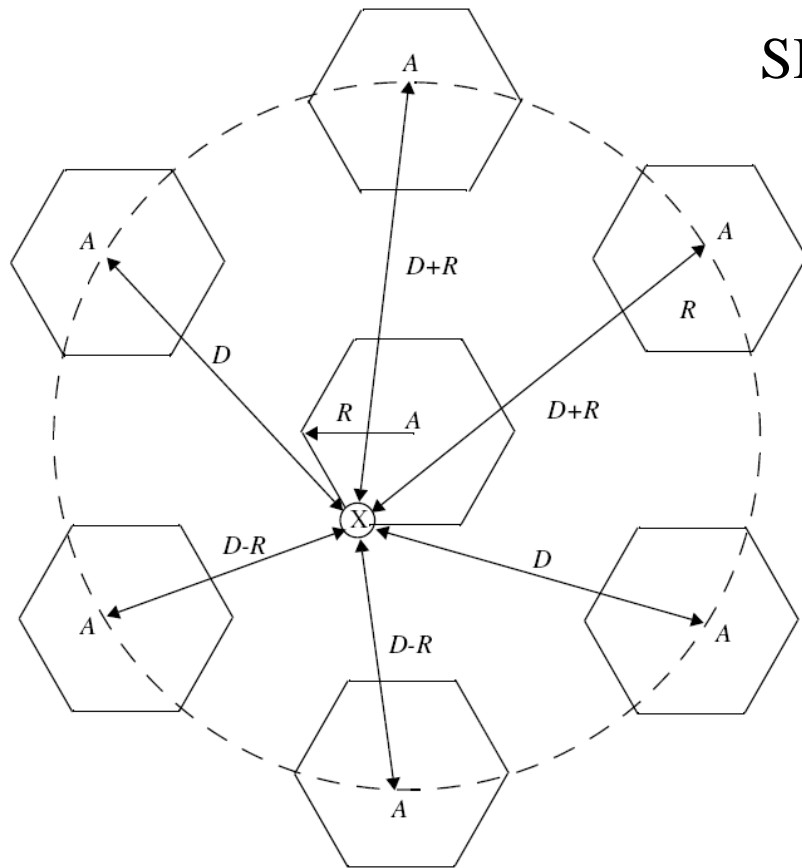
$S$  = total # available duplex radio channels for the system



Frequency reuse with **cluster size  $N$**

# SIR: $N = 7$

Better approximation...



$$\text{SIR} \approx \frac{R^{-\gamma}}{2(D-R)^{-\gamma} + 2(D+R)^{-\gamma} + 2D^{-\gamma}}$$

$$= \frac{1}{2(Q-1)^{-\gamma} + 2(Q+1)^{-\gamma} + 2Q^{-\gamma}}$$

$$\text{Again, } Q = \frac{D}{R} = \sqrt{3N}.$$