1.3 Signal Power Calculation

Monday, November 19, 2012
10:46 AM

Review

$$g(t) = e^{j2\pi f_0 t} \qquad P_g = \langle |e^{j\cdots}|^2 \rangle = \langle 1^2 \rangle = 1$$

$$g(t) = \sum_{k=1}^{n} c_k e^{j2\pi f_k t} \qquad P_g = \sum_{k=1}^{n} |c_k|^2$$

← You will prove this in HW1

(Assume f_k are distinct.)

Signal Power at Tx

Assume that the power of $m(t)$ is P_m.

$$x(t) = m(t) \times \sqrt{2}\cos(2\pi f_c t) \Rightarrow P_x = (\sqrt{2})^2 \frac{1}{2} P_m = P_m = P_t$$

transmitted power

↑ Assume $M(f \pm f_c)$ do not overlap.

$$x(t) \approx \sqrt{P_m} \times \sqrt{2}\cos(2\pi f_c t) \Rightarrow P_x = P_m = P_t$$

For simplification, we will assume $m(t)$ is a constant $\sqrt{P_m}$. This assumption avoid having to think about spectral overlapping of the signal.

You can also try to redo the analysis we did in class with $m(t)$ instead of $\sqrt{2P_m}$.

Signal Power at Rx

$$y(t) = h \times \sqrt{P_m} \times \sqrt{2}\cos\left(2\pi f_c\left(t - \frac{d}{c}\right)\right) = \frac{\alpha}{d}\sqrt{2P_m}\cos\left(2\pi f_c\left(t - \frac{d}{c}\right)\right)$$

fading coefficient (magnitude)

still unknown here

we will use the Friis equation to derive h.

(free space PL model)

$$P_y = h^2 P_t$$

$$\frac{P_y}{P_t} = h^2 \underset{\text{Friis equation}}{=} \left(\frac{\sqrt{G_{Tx} G_{Rx}}\,\lambda}{4\pi d}\right)^2 \Rightarrow h = \underbrace{\frac{\sqrt{G_{Tx} G_{Rx}}\,\lambda}{4\pi}}_{\alpha} \times \frac{1}{d}$$

Summary:

The received signal which travel a distance d away from the transmitter is given by

$$y(t) = \frac{\alpha}{d}\sqrt{2P_m}\cos\left(2\pi f_c\left(t - \frac{d}{c}\right)\right).$$

Here, we assume free-space propagation.

Multipath Reception

In general,

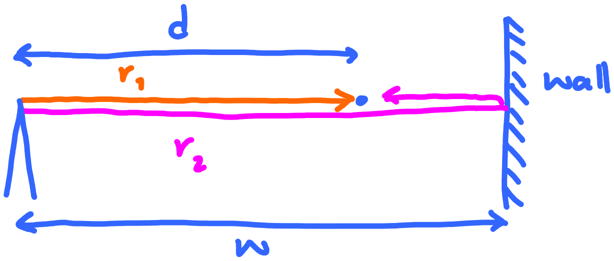
propagation distance for the k^{th} path

$$y(t) = \sum_{k=1}^{n} R_k \frac{\alpha}{r_k}\sqrt{2P_m}\cos\left(2\pi f_c\left(t - \frac{r_k}{c}\right)\right)$$

reflection coefficient

(assume $= 1$ for no reflection
-1 for one reflection.)

Ex. 1.

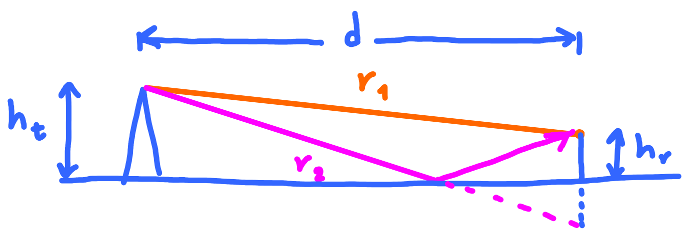


$n = 2$

$r_1 = d$, $R_1 = 1$

$r_2 = 2w - d$, $R_2 = -1$

Ex. 2.



$n = 2$

$r_1 = \sqrt{d^2 + (h_t - h_r)^2}$, $R_1 = 1$

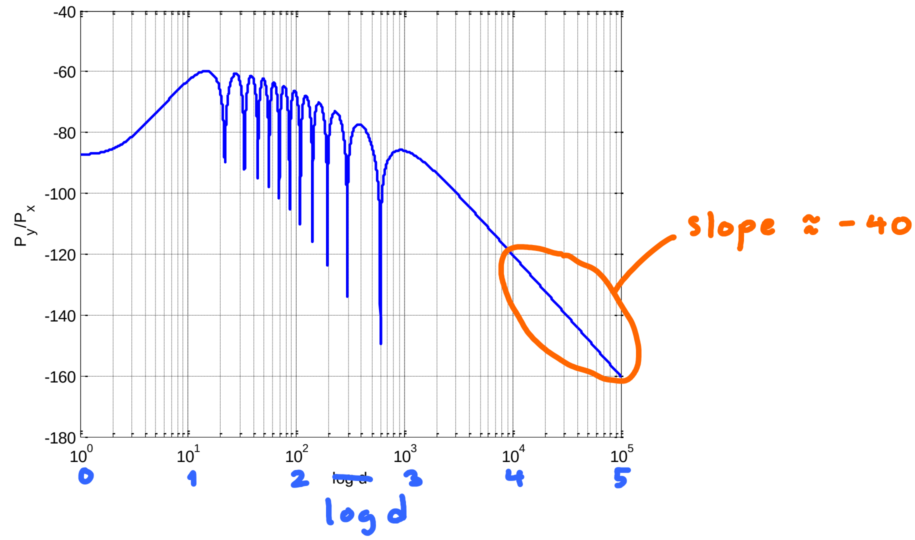
$r_2 = \sqrt{d^2 + (h_t + h_r)^2}$, $R_2 = -1$

Fact: For large d, $r_2 - r_1 \propto \frac{1}{d}$

$$\frac{P_y}{P_x} \propto \frac{1}{d^4}$$

$\gamma = 4$ in the simplified path loss model.

We get $\gamma = 4$ by looking at the plot of $\frac{P_y}{P_x}$



Recall : Simplified path loss model

$$\frac{P_y}{P_x} = \frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

In dB, $10 \log \frac{P_y}{P_x} = 10 \log K d_0^{\gamma} - \underbrace{10\gamma}_{\text{slope}} \log d$

$\Rightarrow \gamma \approx 4$ for the model above.

dB, dBm, dBW

dBmW

dBm, dBW → absolute power → tell signal strength level

dB → relative power → tell the difference (or ratio) between two power levels.

"dB" is used when we compare two power values P_1 and P_2:

$$10 \log_{10} \frac{P_2}{P_1}$$

Rule of Thumb : Double/half power ⇒ add/subtract 3 dB
Ten times/One-tenth power ⇒ add/subtract 10 dB

"dBm" is used when we compare a power with 1 mW :

$$10 \log_{10} \frac{P}{1\,mW}$$

"dBW" is used when we compare a power with 1 W:

$$10 \log_{10} \frac{P}{1\,W}$$

Ex $P = 1\ W = 10 \log \frac{1\,W}{1\,W}\ [dBW] = 0\ [dBW]$

$= 10 \log \frac{1\,W}{1\,mW}\ [dBm] = 30\ [dBm]$

Ex. Now start with $P_1 = 1\ W = 0\ dBW = 30\ dBm$.

Consider $P_2 = 2 \times P_1$. $10 \log_{10} 2 \approx 3\ dB$

Then, P_2 is 3 dB more than P_1

$$P_2 = 2\,W = 3\,[dBW] = 33\ [dBm]$$

Note that we add 3 dB to both 0 dBW and 30 dBm

to get 3 dBW and 33 dBm for the power P_2.

It may look confusing at first that we can directly add 3 dB to quantities in dBW and dBm. It may look less surprising if you remember that dB quantity is simply a (unitless) factor that tells the ratio of the two powers.

3 dB means one power value is twice as much as another.

When you double 1 W, you get 2 W
double 1000 mW, you get 2000 mW.

It does not matter whether you express your power as 1 W or 1000 mW, you simply multiply by 2.

Similarly, in log scale, it does not matter whether you express your power as 0 dBW or 30 dBm, you simply add 3 dB when you double the amount of power.