Q1 Euler's Formula

Thursday, November 11, 2010
2:54 PM

$$\cos A \cos B = \left(e^{jA} + e^{-jA}\right)\left(e^{jB} + e^{-jB}\right) \times \frac{1}{4}$$

$$= \left(e^{j(A+B)} + e^{-j(A+B)} + e^{j(A-B)} + e^{-j(A-B)}\right)\frac{1}{4}$$

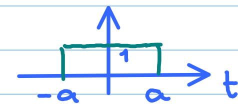
$$\underbrace{e^{j(A+B)} + e^{-j(A+B)}}_{2\cos(A+B)} \qquad \underbrace{e^{j(A-B)} + e^{-j(A-B)}}_{2\cos(A-B)}$$

$$= \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

Q3 Sinc Function and Triangular Signal

Wednesday, July 06, 2011
12:16 PM

We know that



$$2a\,\mathrm{sinc}(2\pi a f) \xrightarrow{\mathcal{F}^{-1}} 1[|t| \leq a]$$

Therefore,

$$\mathrm{sinc}(2\pi a f) \xrightarrow{\mathcal{F}^{-1}} \frac{1}{2a} 1[|t| \leq a].$$

Finally,

$$\mathrm{sinc}^2(2\pi a f) \xrightarrow{\mathcal{F}^{-1}} \frac{1}{2a} 1[|t| \leq a] * \frac{1}{2a} 1[|t| \leq a]$$

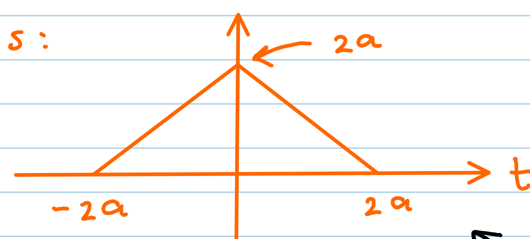
In this question, $2a = 5$.

$$= \frac{1}{4a^2}\left(1[|t| \leq a] * 1[|t| \leq a]\right)$$

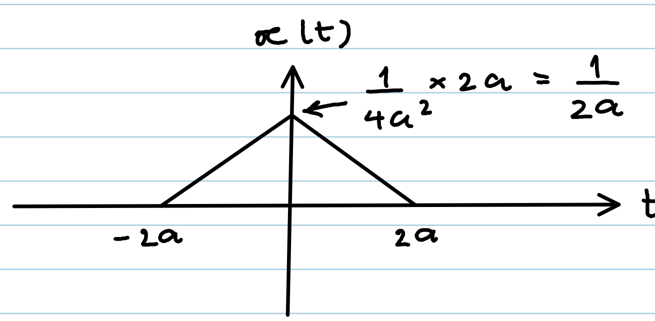
So, we can solve this question if we can find the convolution of $1[|t| \leq a]$ with itself.

This is also discussed in class:

$$1[|t| \leq a] * 1[|t| \leq a] =$$

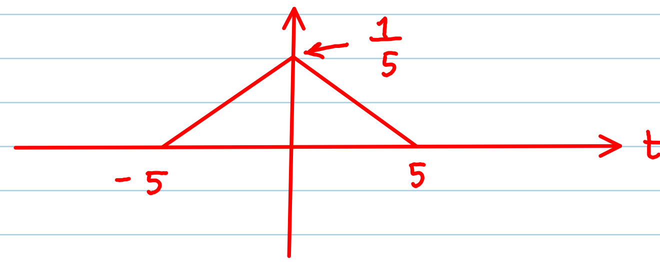


Therefore, the plot of $x(t)$ should be the same as but scaled vertically by a factor of $1/4a^2$:



For us, $a = \frac{5}{2}$. So, $2a = 5$ and the plot of $x(t)$ is

$x(t)$


1/5
t
-5
5

Q4 Manipulation of time

Wednesday, July 06, 2011
12:20 PM

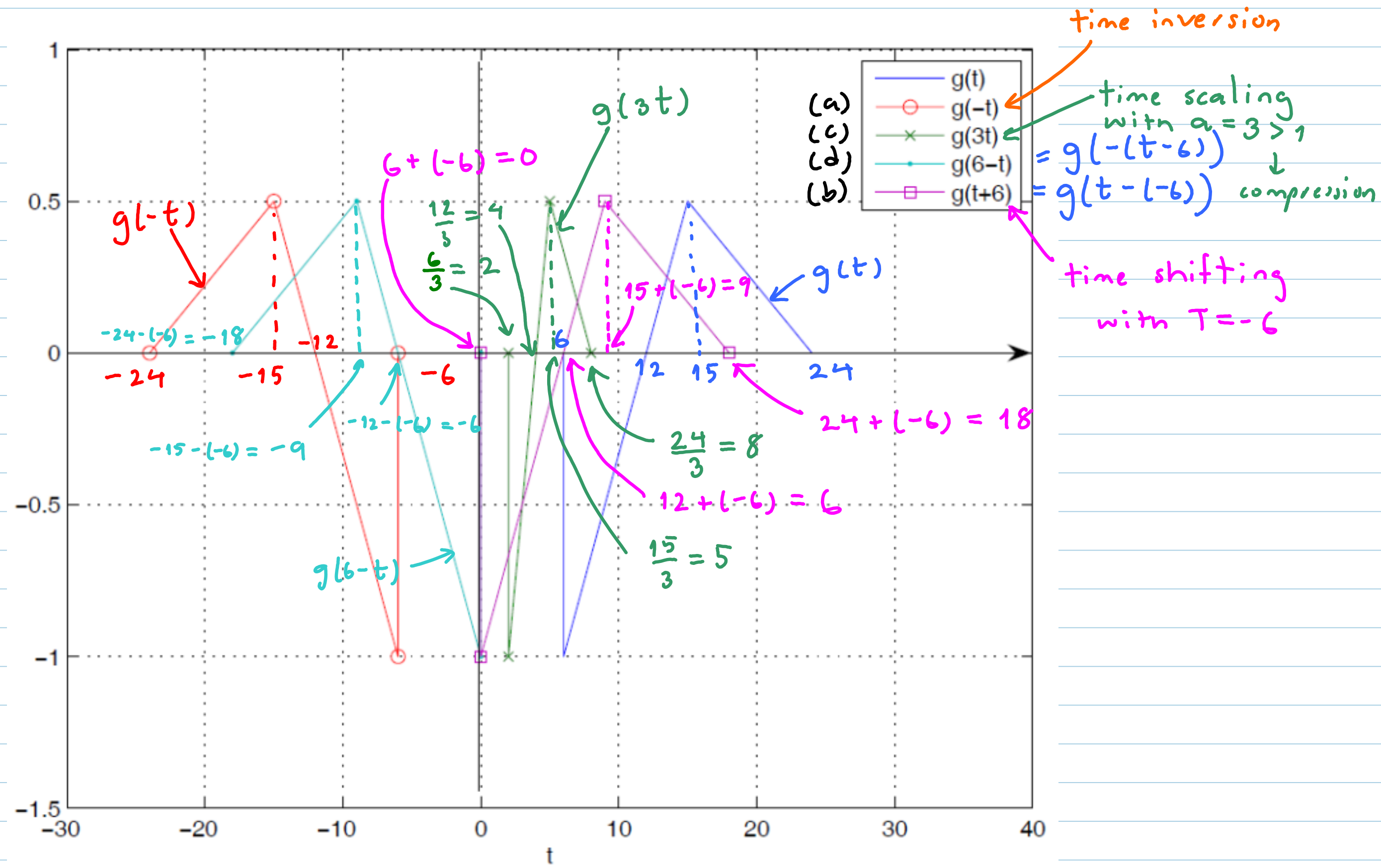
First, we review some useful signal operations

Time shifting : $g(t-T)$ represents $g(t)$ time-shifted by T.
If T is positive, the shift is to the right (delay).
If T is negative, the shift is to the left.

Time scaling : If $g(t)$ is compressed in time by a factor a $(a>1)$, the resulting signal is $g(at)$.
If $0<a<1$, the scaling is expansion.

Time inversion (Time reversal)
: $g(-t)$ is the mirror image of $g(t)$ about the vertical axis.

All the signals are plotted below



The tricky one would be $g(6-t)$.

There are two ways to think about it

time inversion time shift, $T=6$

There are two ways to think about it

① $g(t) \xrightarrow{\text{time inversion}} g(-t) \xrightarrow{\text{time shift, } T=6} g(-(t-6))$

$g(-t)$: mirror image about the vertical axis

$g(-(t-6))$: shift to the right by 6

② $g(t) \xrightarrow{\text{time shift, } T=-6} g(t+6) \xrightarrow{\text{time inversion}} g(-t+6)$

$g(t+6)$: shift to the left by 6

$g(-t+6)$: mirror image of $g(t+6)$ about the vertical axis

Q5 Using Properties of FT

Wednesday, July 06, 2011
1:11 PM

(b) Note that $g_1(t) = g(-t)$.

Recall that $x(at) \xrightarrow{\mathcal{F}} \frac{1}{|a|} X\left(\frac{f}{a}\right)$.

Here, $a = -1$.

Therefore, $G_1(f) = \frac{1}{|-1|} G\left(\frac{f}{-1}\right) = \frac{1}{(2\pi f)^2}\left(e^{-j2\pi f} + j2\pi f e^{-j2\pi f} - 1\right)$

(c) Note that $g_2(t) = g(t-1) + g_1(t-1)$

$$\Rightarrow \quad G_2(f) = e^{-j2\pi f} G(f) + e^{-j2\pi f} G_1(f)$$

$$= \frac{e^{-j\omega}}{\omega^2}\left(e^{j\omega} - j\omega e^{j\omega} - 1 + e^{-j\omega} + j\omega e^{-j\omega} - 1\right)$$

$$= \frac{e^{-j\omega}}{\omega^2}\left(2\cos(\omega) - j\omega(2j)\sin\omega - 2\right)$$

$$= \frac{2e^{-j2\pi f}}{(2\pi f)^2}\left(\cos(2\pi f) + 2\pi f\sin(2\pi f) - 1\right)$$

(d) Note that $g_3(t) = g(t-1) + g_1(t+1)$

$$\Rightarrow \quad G_3(f) = e^{-j2\pi f} G(f) + e^{j2\pi f} G_1(f)$$

$$= \frac{1}{\omega^2}\left(1 - \cancel{j\omega} - e^{-j\omega} + 1 + \cancel{j\omega} - e^{j\omega}\right)$$

$$= \frac{1}{\omega^2}\left(2 - 2\cos(\omega)\right) = \frac{2}{\omega^2}(1 - \cos\omega)$$

Recall that

$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

$\parallel \qquad \parallel$

$1 - \sin^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$

$\Rightarrow \cos 2A = 1 - 2\sin^2 A$

$$= \frac{2}{\omega^2} 2\sin^2\left(\frac{\omega}{2}\right) = \left(\frac{\sin\left(\frac{\omega}{2}\right)}{\omega/2}\right)^2 = \text{sinc}^2\left(\frac{\omega}{2}\right)$$

$1-\cos 2A = 2\sin^2 A$

$$= \mathrm{sinc}^2(\pi f)$$

(e) Note that $g_4(t) = g(t-\tfrac{1}{2}) + g_1(t+\tfrac{1}{2})$.

$$\Rightarrow \quad G_4(f) = e^{-j\omega/2} G(f) + e^{j\omega/2} G_1(f)$$

use part (b)

$$= e^{-j\omega/2} \frac{1}{\omega^2}\left(e^{j\omega} - j\omega e^{j\omega} - 1\right)$$

$$+ e^{j\omega/2} \frac{1}{\omega^2}\left(e^{-j\omega} + j\omega e^{-j\omega} - 1\right)$$

$$= \frac{1}{\omega^2}\left(\cancel{e^{j\frac{\omega}{2}}} - j\omega e^{j\frac{\omega}{2}} - \cancel{e^{-j\frac{\omega}{2}}} + \cancel{e^{-j\frac{\omega}{2}}} + j\omega e^{-j\frac{\omega}{2}} - \cancel{e^{j\frac{\omega}{2}}}\right)$$

$$= \frac{-j}{\omega}\left(e^{j\omega/2} - e^{-j\omega/2}\right) = \frac{(-j)}{\omega}(2j)\sin(\omega/2)$$

$$= \frac{\sin(\omega/2)}{\omega/2} = \mathrm{sinc}\left(\frac{\omega}{2}\right) = \mathrm{sinc}(\pi f)$$

(f) Note that $g_5(t) = 1.5\, g\left(\tfrac{1}{2}(t-2)\right)$

$$\Rightarrow \quad G_5(f) = 1.5 \times \frac{1}{1/2} G\left(\frac{f}{1/2}\right) e^{-j2\omega}$$

$$= 3\, G(2f)\, e^{-j}$$

$$= 3 \times \frac{1}{(2\pi 2f)^2}\left(e^{j2\omega} - j2\omega e^{j2\omega} - 1\right) e^{-j2\omega}$$

$$= \frac{3}{4\omega^2}\left(1 - 2j\omega - e^{-2j\omega}\right)$$

$$= \frac{3}{4(2\pi f)^2}\left(1 - j4\pi f - e^{-j4\pi f}\right)$$

Q6 Sq MODEM

Thursday, July 14, 2011
2:22 PM

$x(f) = A_c M(f)$

(a) $x(t) = A_c m(t) \xrightarrow{\mathcal{F}}$ So, $X(f)$ is also bandlimited to B.

$u(t) = x(t) + \sqrt{2}\cos(\omega_c t)$ $\quad \omega_c = 2\pi f_c$

$v(t) = u^2(t) = \left(x(t) + \sqrt{2}\cos(\omega_c t)\right)^2$

$= x^2(t) + 2\sqrt{2}\,x(t)\cos(\omega_c t) + \underbrace{2\cos^2(\omega_c t)}_{1+\cos(2\omega_c t)}$

$= \underbrace{\left(1 + x^2(t)\right)}_{\text{BPF} \to 0} + 2\sqrt{2}\,x(t)\cos\omega_c t + \underbrace{\cos(2\omega_c t)}_{\text{BPF} \to 0}$

Note1: $x^2(t) \xrightarrow{\mathcal{F}} X(f) * X(f)$

So, $x^2(t)$ is bandlimited to $2B$

Because $f_c \gg B$, the spectrum of $x^2(t)$ will <u>not</u> be in the passband of the BPF which centers around f_c.

Note 2: The term $\cos(2\omega_c t)$ is at frequency $2 \times f_c$ which again is outside the passband.

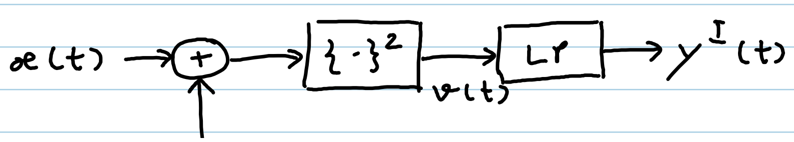
$y(t) = \text{BPF}\{v(t)\}$

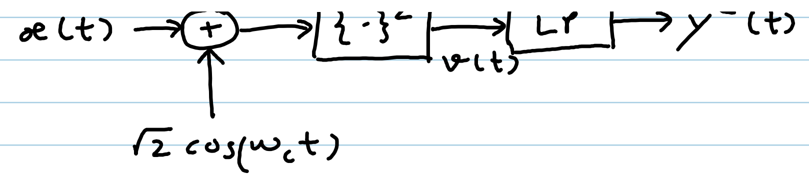
$= 2\sqrt{2}\,x(t)\cos\omega_c t$

$= 2\sqrt{2}\,A_c\,m(t)\cos\omega_c t$

(b) Assume

$x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$





From the above figure,

$$v(t) = \left(x(t) + \sqrt{2}\cos(\omega_c t)\right)^2$$

$$= 2\cos^2(\omega_c t)\left(A_c m(t) + 1\right)^2$$

$$= \left(1 + \cos(2\omega_c t)\right)\underbrace{\left(A_c^2 \underbrace{m^2(t)}_{\substack{\text{spectrum}\\ \text{is from}\\ [-2B,\,2B]}} + 1 + 2A_c \underbrace{m(t)}_{\substack{\text{spectrum}\\ \text{is from}\\ [-B,\,B]}}\right)}_{g(t)}$$

$$= g(t) + \underbrace{g(t)\cos(2\omega_c t)}_{\xrightarrow{\text{LPF}}\ 0}$$

Note 1: We know that $g(t)$ is band limited to $[-2B, 2B]$ because all of its terms are band limited to $[-2B, 2B]$. So, only some parts of it will pass through the LPF.

Note 2: $g(t)\cos(2\omega_c t)$ is centered @ $2f_c$ and therefore will not pass thought the LPF.

$$y^I(t) = \text{LPF}\{v(t)\}$$

$$= \text{LPF}\{g(t)\}$$

$$= 1 + 2A_c m(t) + \text{LPF}\{A_c^2 m^2(t)\}$$

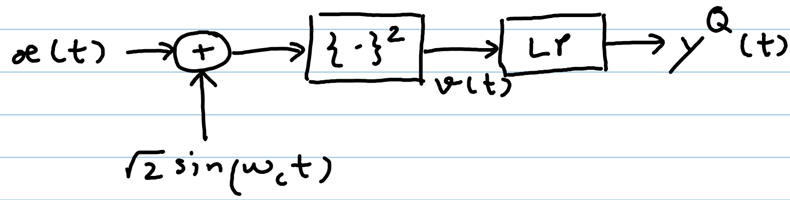
This term has spectrum beyond IW so, only a portion of it will pass through the LPF.

$y^I(t)$ is not proportional to $m(t)$.

Hence, this block diagram does not work as a demodulator.

(c) Assume

$x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$ as in part (b).



We then have

$$
\begin{aligned}
v(t) &= \left(x(t) + \sqrt{2}\sin(\omega_c t)\right)^2 \\
&= 2\left(A_c m(t)\cos(\omega_c t) + \sin(\omega_c t)\right)^2 \\
&= 2\left(A_c^2 m^2(t)\cos^2(\omega_c t) + A_c m(t)\cos(\omega_c t)\sin(\omega_c t) + \sin^2(\omega_c t)\right) \\
&= 2\left(A_c^2 m^2(t)\cos^2(\omega_c t) + \sin^2(\omega_c t)\right) + A_c m(t)\sin(2\omega_c t) \\
&= 2\left(\left(A_c^2 m^2(t) - 1\right)\cos^2(\omega_c t) + 1\right) + A_c m(t)\sin(2\omega_c t) \\
&= 2 + \left(A_c^2 m^2(t) - 1\right)\left(1 + \cos(2\omega_c t)\right) + A_c m(t)\sin(2\omega_c t)
\end{aligned}
$$

LPF 0 LPF 0

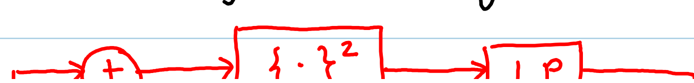
$$
\begin{aligned}
y^Q(t) &= 2 + \mathrm{LPF}\{A_c^2 m^2(t)\} - 1 \\
&= \mathrm{LPF}\{A_c^2 m^2(t)\} + 1
\end{aligned}
$$

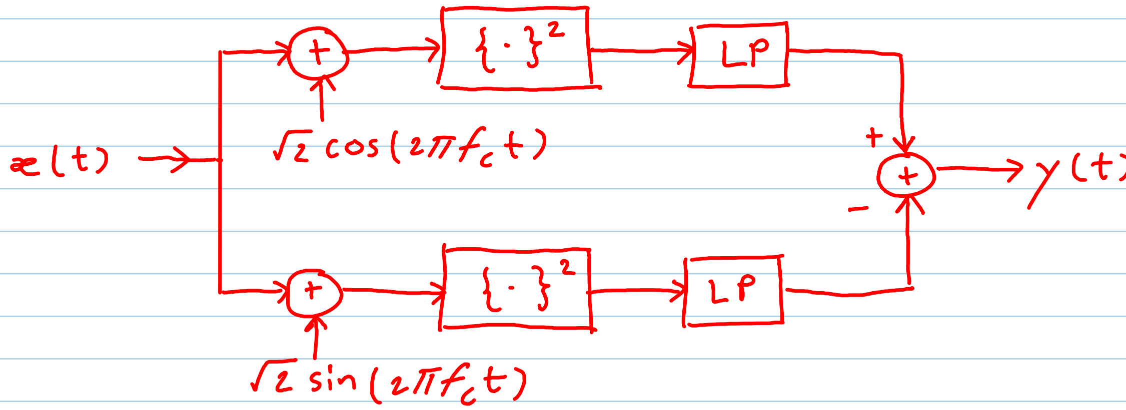
(d) Observe that

$y^I(t) - y^Q(t) = 2A_c m(t)$ which is the desired output of a successful DSB-SC demodulator.

from (b) from (c)

Hence, the following block diagram would work:




x(t)
√2 cos(2πf_c t)
√2 sin(2πf_c t)
{·}²
{·}²
LP
LP
+
−
y(t)

Q7

Tuesday, June 28, 2011
9:06 AM

Consider $x(t) \xrightarrow{\mathcal{F}} X(f)$.

(a)

Let $y(t) = x^*(t)$. We want to find $Y(f)$.

First, recall that $X(f) = \int_{-\infty}^{\infty} x(t) e^{j2\pi f t} dt$.

Hence, $Y(f) = \int_{-\infty}^{\infty} x^*(t) e^{j2\pi f t} dt = \Big(\underbrace{\int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt}_{X(-f)} \Big)^*$

$= \left(X(-f)\right)^* = X^*(-f)$

(b)

Let $y(t) = \mathrm{Re}\{x(t)\}$.

From the hint, we first note that $x(t) + x^*(t) = 2\,\mathrm{Re}\{x(t)\}$.

Hence, $y(t) = \mathrm{Re}\{x(t)\} = \frac{1}{2}\left(x(t) + x^*(t)\right)$ and

$Y(f) = \frac{1}{2}\left(X(f) + \mathcal{F}\{x^*(t)\}\right) = \frac{1}{2}\left(X(f) + X^*(-f)\right)$

From part (a)

Remarks: (1) The expression for $Y(f)$ above is similar to $\mathrm{Re}\{X(f)\}$ but they are not the same.

Compare:

$\mathrm{Re}\{X(f)\} = \frac{1}{2}\left(X(f) + X^*(f)\right)$, and

$Y(f) = \mathcal{F}\{\mathrm{Re}\{x(t)\}\} = \frac{1}{2}\left(X(f) + X^*(-f)\right)$.

$$Y(f) = \mathcal{F}\{\mathrm{Re}\{x(t)\}\} = \frac{1}{2}\left(X(f) + X^*(-f)\right).$$

(2) When $x(t)$ is real-valued,

$$y(t) = \mathrm{Re}\{x(t)\} = x(t), \quad \text{and}$$

$$Y(f) = \mathcal{F}\{y(t)\} = \mathcal{F}\{x(t)\} = X(f)$$

Let's check whether $Y(f) = X(f)$ if we use

Recall that for real-valued $x(t)$,

$$X(-f) = X^*(f)$$

So, $Y(f) = \frac{1}{2}\left(X(f) + X^*(-f)\right) = \frac{1}{2}\left(X(f) + \left(X^*(f)\right)^*\right)$

$$= X(f). \quad \checkmark$$

(3) Let's try another check.

Because $y(t)$ is defined as $\mathrm{Re}\{x(t)\}$, we know that $y(t)$ will always be real-valued.

Hence, it must also satisfy the conjugate symmetry property:

$$Y(-f) = Y^*(f).$$

So, let's try plugging "$-f$" into our expression for $Y(f)$:

$$Y(f) = \frac{1}{2}\left(X(f) + X^*(-f)\right)$$

This gives

$$Y(-f) = \frac{1}{2}\left(X(-f) + X^*(f)\right)$$

of course,

$$Y^*(f) = \frac{1}{2}\left(X^*(f) + X(-f)\right)$$

Therefore, $Y(-f) = Y^*(f)$ as expected.

Q8 Complex-valued Representation of QAM

Tuesday, June 28, 2011
9:34 AM

(a) $x_b(t) \xrightarrow{\mathcal{F}} X_b(f)$

By the freq.-shift property of Fourier transform,

$$\underbrace{e^{j2\pi f_c t} x_b(t)} \xrightarrow{\mathcal{F}} X_b(f-f_c)$$

Call this $g(t)$. Then, $G(f) = X_b(f-f_c)$

Recall, from the previous problem that

$$\text{Re}\{g(t)\} \xrightarrow{\mathcal{F}} \frac{1}{2}\left(G(f) + G^*(-f)\right).$$

Note that $x_p(t) = \sqrt{2}\,\text{Re}\{g(t)\}$.

Hence,

$$X_p(f) = \sqrt{2} \times \frac{1}{2}\left(G(f) + G^*(-f)\right)$$

$$= \frac{1}{\sqrt{2}}\left(X_b(f-f_c) + X_b^*(-f-f_c)\right)$$

(b) By the freq.-shift property of FT,

$$x_p(t)\, e^{-j2\pi f_c t} \xrightarrow{\mathcal{F}} X_p(f-(-f_c)) = X_p(f+f_c)$$

$$= \frac{1}{\sqrt{2}}\left(X_b(f+f_c-f_c) + X_b^*(-(f+f_c)-f_c)\right)$$

$$= \frac{1}{\sqrt{2}}\left(X_b(f) + X_b^*(-(f+2f_c))\right)$$

Therefore,

$$\sqrt{2}\, x_p(t)\, e^{-j2\pi f_c t} \xrightarrow{\mathcal{F}} X_b(f) + X_b^*(-(f+2f_c))$$

$$\text{LPF}\left\{ \quad \right\} \xrightarrow{\mathcal{F}} X_b(f) + 0 = X_b(f)$$

LPF

Problem 9

a.

$$P(f)=\int_{-\infty}^{\infty} p(t)e^{-j2\pi ft}dt=\int_{0}^{T} Ae^{-j2\pi ft}dt=A\frac{1}{-j2\pi f}e^{-j2\pi ft}\Big|_0^T$$

$$=A\frac{1}{-j2\pi f}\left(e^{-j2\pi fT}-1\right)=\frac{A}{j2\pi f}\left(1-e^{-j2\pi fT}\right)$$

b. We start with $x(t)=\sum_{k=0}^{\ell-1} m_k p(t-kT)\xrightarrow{\mathcal{F}} X(f)=P(f)\sum_{k=0}^{\ell-1} m_k e^{-j2\pi fkT}$.

Hence,

$$X(f)=P(f)\left(m_0+m_1e^{-j1\pi fT}+m_2e^{-j2\pi fkT}+m_3e^{-j2\pi fkT}\right);\ell=4$$

$$=\frac{A}{j2\pi f}(1-z)\left(m_0+m_1z+m_2z^2+m_3z^3\right);\ z=e^{-j1\pi fT}$$

$$=\frac{A}{j2\pi f}(1-z)\left(1-z+z^2+z^3\right)$$

$$=\frac{A}{j2\pi f}\left(1-2z-2z^2-z^4\right)$$

$$=\frac{A}{j2\pi f}\left(1-2e^{-j1\pi fT}-2e^{-j2\pi fT}-e^{-j4\pi fT}\right)$$

c. First, we find

$$P(0)=\int_{-\infty}^{\infty} p(t)e^{-j2\pi 0t}dt=\int_{-\infty}^{\infty} p(t)dt=AT$$

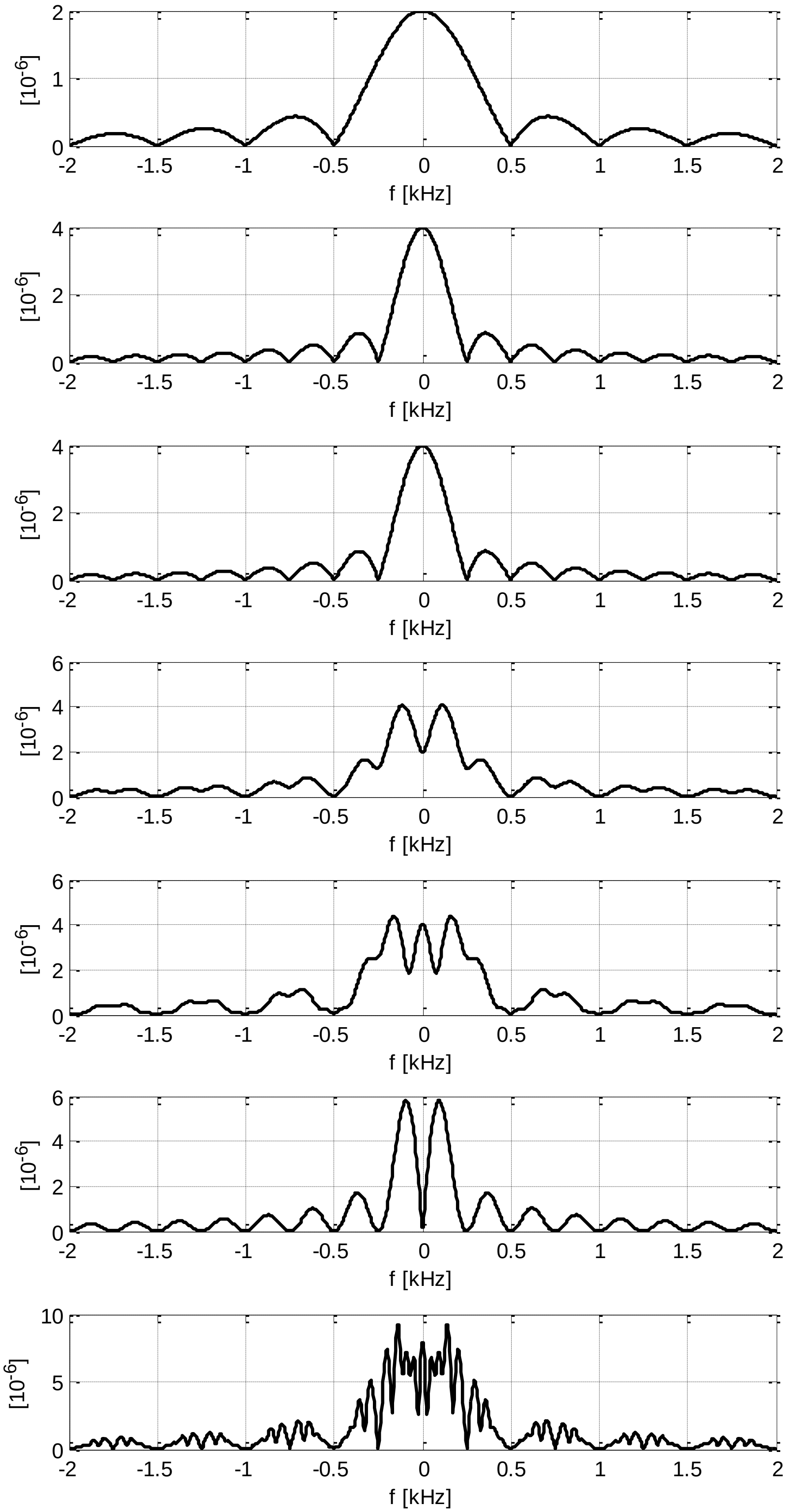
Then

$$X(0)=P(0)\sum_{k=0}^{\ell-1} m_k e^{-j2\pi 0kT}=P(0)\sum_{k=0}^{\ell-1} m_k=AT\sum_{k=0}^{\ell-1} m_k$$

After plugging in the numbers, we have

$$X(0)\ =\ 2,\ 4,\ 4,\ 2,\ 4,\ 0,\ 8\quad \times 10^{-6}\quad [\mathrm{V/Hz}]$$

d. All the plots are shown on the next page.



2
1
0
[10-6]
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]
4
2
0
[10-6]
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]
4
2
0
[10-6]
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]
6
4
2
0
[10-6]
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]
6
4
2
0
[10-6]
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]
6
4
2
0
[10-6]
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]
10
5
0
[10-6]
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]

Q10

Wednesday, July 27, 2011

8:23 PM

(a) We know that

$$1[|t| \le a] \xrightarrow{\mathcal{F}} 2a\,\mathrm{sinc}(2\pi f a)$$

So,

$$1[|t| \le a] = \int_{-\infty}^{\infty} 2a\,\mathrm{sinc}(2\pi f a)\, e^{j2\pi f t}\, df$$

Inverse transform

For $a>0$, we have

$$\int_{-\infty}^{\infty} \mathrm{sinc}(2\pi f a)\, e^{j2\pi f t}\, df = \frac{1}{2a}\, 1[|t| \le a]$$

Setting $t=0$ leads to

$$\int_{-\infty}^{\infty} \mathrm{sinc}(2\pi f a)\, df = \frac{1}{2a} = \frac{1}{\cancel{2} \times \frac{\sqrt{5}}{\cancel{2}\pi}} = \frac{\pi}{\sqrt{5}}$$

Here, $2\pi a = \sqrt{5} \Rightarrow a = \frac{\sqrt{5}}{2\pi}$

(b) Note first that $2\,\mathrm{sinc}(2\pi f) \xrightarrow{\mathcal{F}^{-1}} 1[|t| \le 1]$ $(a=1)$

By the time-shift property,

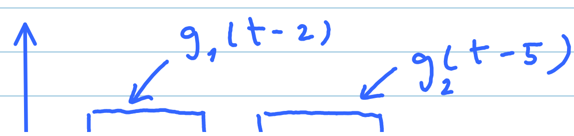
$$e^{-j2\pi f t_0}\, 2\,\mathrm{sinc}(2\pi f) \xrightarrow{\mathcal{F}^{-1}} 1[|t-t_0| \le 1]$$

By Parseval's theorem

$$\int_{-\infty}^{\infty} \left(e^{-j2\pi f t_1} G_1(f)\right)\left(e^{-j2\pi f t_2} G_2(f)\right)^* df =$$

$$= \int_{-\infty}^{\infty} g_1(t-t_1)\, g_2^*(t-t_2)\, dt$$

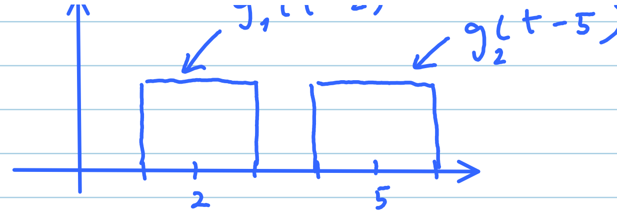
Here, $g_1(t) = g_2(t) = 1[|t| \le 1]$



$t_1 = 2$, $t_2 = 5$

Here, $g_1(t) = g_2(t) = 1[|t| \le 1]$

$t_1 = 2, \quad t_2 = 5$



No overlap, so the integral is 0.

Alternatively, we can first simplify the integral to

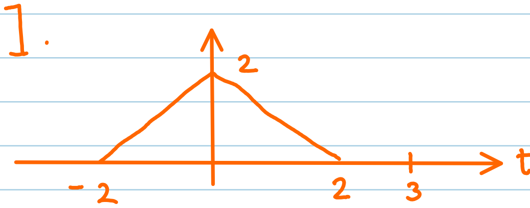
$$\int_{-\infty}^{\infty} e^{j2\pi f(t_2 - t_1)} G_1(f) G_2(f)\, df$$

This is then the inverse Fourier transform of $G_1(f)G_2(f)$ evaluated at $t = (t_2 - t_1)$.

The inverse Fourier transform is given by $g_1(t) * g_2(t)$.

Again, $g_1(t) = g_2(t) = 1[|t| \le 1]$.

So, $g_1(t) * g_2(t) =$



Here, $t_2 - t_1 = 5 - 2 = 3$. So, the integral is 0.

(c) $\text{sinc}(\underbrace{2\pi a}_{=c} f) \xrightarrow{\mathcal{F}^{-1}} \frac{1}{2a} 1[|t| \le a]$.

$\Downarrow$

$a = \frac{c}{2\pi}$

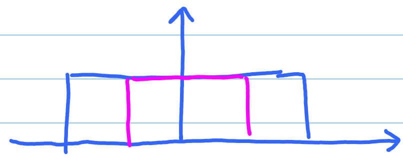
$\text{sinc}(cf) \xrightarrow{\mathcal{F}^{-1}} \frac{\pi}{c} 1\left[|t| \le \frac{c}{2\pi}\right]$

Again, by Parseval's theorem,

$$\int_{-\infty}^{\infty} \text{sinc}(c_1 f)\, \text{sinc}^*(c_2 f)\, df = \int_{-\infty}^{\infty} \frac{\pi}{c_1} 1\left[|t| \le \frac{c_1}{2\pi}\right] \frac{\pi}{c_2} 1\left[|t| \le \frac{c_2}{2\pi}\right] dt$$

$$= \frac{\cancel{\pi^2}}{c_1 c_2} \times \frac{\min\{c_1, c_2\}}{\cancel{\pi}} = \frac{\pi}{c_1 c_2} \min\{c_1, c_2\}.$$

$$= \frac{\pi^{\cancel{2}}}{c_1 c_2} \times \frac{\min\{c_1, c_2\}}{\cancel{\pi}} = \frac{\pi}{c_1 c_2} \min\{c_1, c_2\}.$$



Here, $c_1 = \sqrt{5}$, $c_2 = \sqrt{7}$.

So, the integral is $\frac{\pi}{\cancel{\sqrt{5}}\sqrt{7}} \cancel{\sqrt{5}} = \frac{\pi}{\sqrt{7}}$

Alternatively, the integral is the inverse Fourier transform of $\mathrm{sinc}(c_1 f)\,\mathrm{sinc}(c_2 f)$ evaluated at $t = 0$.

same calculation

(d) $\mathrm{sinc}(cf) \xrightarrow{\mathcal{F}^{-1}} \frac{\pi}{c} 1\left[|t| \le \frac{c}{2\pi}\right]$

$\downarrow c = \pi$

$\mathrm{sinc}(\pi f) \xrightarrow{\mathcal{F}^{-1}} 1\left[|t| \le \frac{1}{2}\right]$

$\mathrm{sinc}(\pi(f - f_0)) \xrightarrow{\mathcal{F}^{-1}} e^{j2\pi f_0 t}\, 1\left[|t| \le \frac{1}{2}\right]$

By Parseval's theorem, the integral is the same as

$$\int_{-\infty}^{\infty} e^{j2\pi f_1 t}\, 1\left[|t| \le \tfrac{1}{2}\right] e^{-j2\pi f_2 t}\, 1\left[|t| \le \tfrac{1}{2}\right] dt$$

$$= \int_{-1/2}^{1/2} e^{j2\pi(f_1 - f_2)t}\, dt = \frac{1}{j2\pi(f_1 - f_2)} e^{j2\pi(f_1 - f_2)t}\Big|_{-1/2}^{1/2}$$

$$= \frac{1}{j2\pi(f_1 - f_2)} \left(e^{j2\pi(f_1 - f_2)\frac{1}{2}} - e^{-j2\pi(f_1 - f_2)\frac{1}{2}} \right)$$

$$= \frac{\sin(\pi(f_1 - f_2))}{\pi(f_1 - f_2)} = \mathrm{sinc}(\pi(f_1 - f_2))$$

If $f_1 - f_2$ is an integer, then the integral is 0.

Here, $f_1 - f_2 = 5 - \frac{7}{2} = \frac{3}{2}$.

So, the integral is $\dfrac{\sin\left(\frac{3}{2}\pi\right)}{\frac{3}{2}\pi} = \dfrac{-1}{\frac{3}{2}\pi} = -\dfrac{2}{3\pi}$.