ECS 455: Principles of Communications **2011/2**

HW 1 — Due: January 31

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. In class, we have seen how to use the Euler's formula to show that

$$\cos^2 x = \frac{1}{2}\left(\cos\left(2x\right) + 1\right).$$

For this question, use similar technique to show that

$$\cos A \cos B = \frac{1}{2}\left(\cos\left(A+B\right) + \cos\left(A-B\right)\right).$$

Problem 2. Listen to the Fourier's Song (Fouriers_Song.mp3) which can be downloaded from

http://sethares.engr.wisc.edu/mp3s/Fouriers_Song.mp3

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Which properties of the Fourier Transform can you recognize from the song? List them here.

Problem 3. Derive and plot the signal $x(t)$ whose Fourier transform is given by

$$X\left(f\right) = \operatorname{sinc}^2\left(5\pi f\right) = \left(\frac{\sin\left(5\pi f\right)}{\left(5\pi f\right)}\right)^2.$$

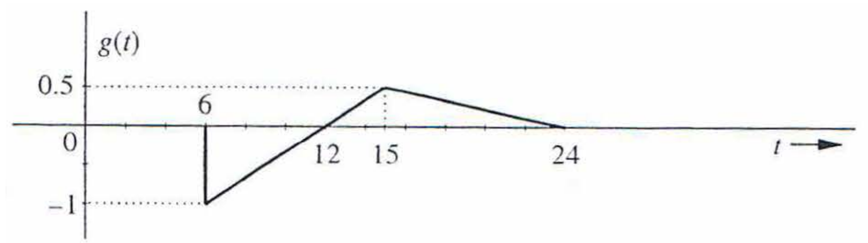

Figure 1.1: Problem 4

Problem 4. For the signal $g(t)$ shown in Figure 1.1, sketch the signals:

(a) $g(-t)$

(b) $g(t+6)$

(c) $g(3t)$

(d) $g(6-t)$.

Problem 5. The Fourier transform of the triangular pulse $g(t)$ in Figure 1.2a is given as

$$G(f) = \frac{1}{(2\pi f)^2}\left(e^{j2\pi f} - j2\pi f e^{j2\pi f} - 1\right)$$

Using this information, and the time-shifting and time scaling properties, find the Fourier transforms of the signals shown in Figure 1.2b, c, d, e, and f.

Problem 6. ***Square-modulator and square-demodulator*** for DSB-SC.

(a) Let $x(t) = A_c m(t)$ where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Consider the block diagram shown in Figure 1.3.

Assume $f_c \gg B$ and

$$H_{BP}(f) = \begin{cases} 1, & |f - f_c| \le B \\ 1, & |f + f_c| \le B \\ 0, & \text{otherwise.} \end{cases}$$

The block labeled "$\{\cdot\}^2$" has output $v(t)$ that is the square of its input $u(t)$:

$$v(t) = u^2(t).$$

Find $y(t)$.

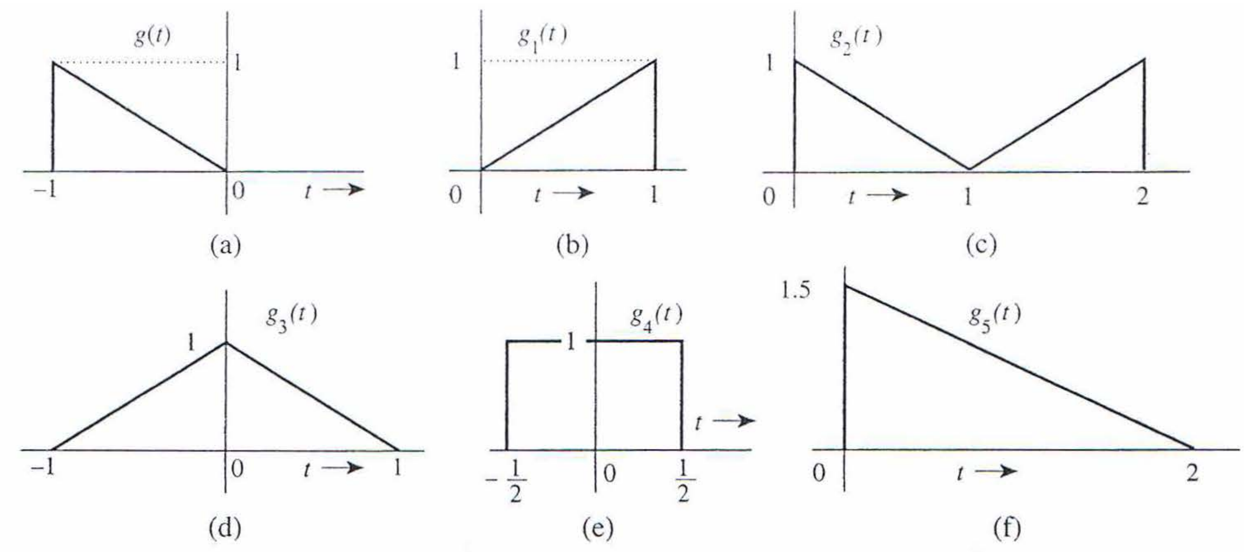

Figure 1.2: Problem 5

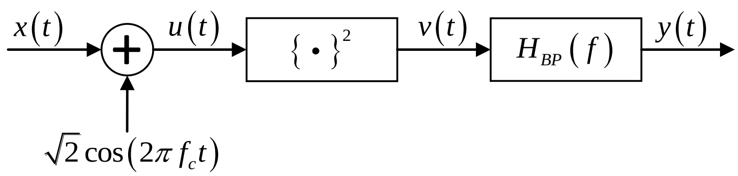

Figure 1.3: Block diagram for Problem 6a

(b) The block diagram in part (a) provides a nice implementation of a modulator because it may be easier to build a squarer than to build a multiplier. Based on the successful use of a squaring operation in the modulator, we decide to use the same squaring operation in the demodulator. Let

$$x(t) = A_c m(t)\sqrt{2}\cos(2\pi f_c t)$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 1.4.

Use

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^I(t)$. Does this block diagram work as a demodulator; that is, is $y^I(t)$ proportional to $m(t)$?

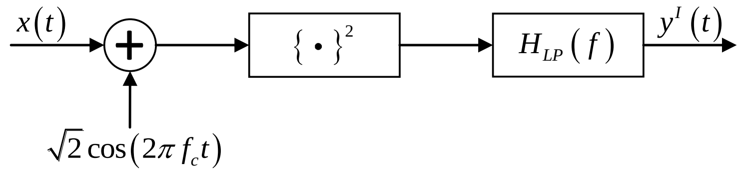

Figure 1.4: Block diagram for Problem 6b

(c) Due to the failure in part (b), we have to think hard and it seems natural to consider also the block diagram with cos replaced by sin. Let

$$x(t) = A_c m(t)\sqrt{2}\cos(2\pi f_c t)$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$ as in part (b). Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 1.5.

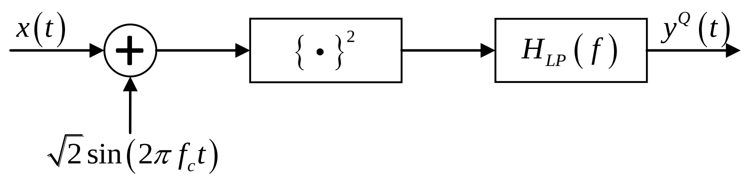

Figure 1.5: Block diagram for Problem 6c

As in part (b), use

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^Q(t)$.

(d) Use the results from parts (b) and (c). Draw a block diagram of a ***successful*** DSB-SC demodulator using squaring operations instead of multipliers.

Problem 7. Consider a complex-valued signal $x(t)$ whose Fourier transform is $X(f)$.

(a) Find and simplify the Fourier transform of $x^*(t)$.

(b) Find and simplify the Fourier transform of $\mathrm{Re}\{x(t)\}$.

- Hint: $x(t) + x^*(t) = ?$

Problem 8. Consider a (complex-valued) baseband signal $x_b(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} X_b(f)$ which is band-limited to B, i.e., $|X_b(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

(a) The passband signal $x_p(t)$ is given by

$$x_p(t) = \sqrt{2}\,\mathrm{Re}\left\{e^{j2\pi f_c t} x_b(t)\right\}.$$

Find and simplify the Fourier transform of $x_p(t)$.

(b) Find and simplify

$$\mathrm{LPF}\left\{\sqrt{2}\left(\underbrace{\sqrt{2}\,\mathrm{Re}\left\{e^{j2\pi f_c t} x_b(t)\right\}}_{x_p(t)}\right) e^{-j2\pi f_c t}\right\}.$$

Assume that the frequency response of the LPF is given by

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Problem 9.

(a) Give a simplified expression for the Fourier transform $P(f)$ of a waveform $p(t)$ when

$$p(t) = \begin{cases} A, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases}$$

(b) A message $m = (m[0], m[1], m[2], m[3]) = (1, -1, 1, 1)$ is sent via

$$x(t) = \sum_{k=0}^{\ell-1} m[k]\, p(t - kT),$$

where ℓ is the length of m.

Find a simplified expression for the Fourier transform $X(f)$ of the waveform $x(t)$.

(c) Assume $T = 2$ [ms] and $A = 1$ [mV]. For $X(f)$ generated by m given below, analytically evaluate $X(0)$.

(i) $m = (1)$;

(ii) $m = (1, 1)$

(iii) $m = (1, 1, 0, 0)$

(iv) $m = (1, 1, -1)$

(v) $m = (1, 1, -1, 1)$

(vi) $m = (1, 1, -1, -1)$

(vii) $m = (1, 1, -1, 1, -1, -1, 1, 1, 1, -1, 1, 1)$

(d) When we know how to find $X(f)$ analytically, we will now use its expression to plot $|X(f)|$ in `MATLAB`. With the help of the provided function `FTofManyShiftedRect.m`, you may run `HW1_Q9.m` to plot $|X(f)|$ from part (b).

Modify the code in `HW1_Q9.m` to plot $|X(f)|$ for the m given in part (c).

(e) What did you learn from the plots in part (d)?

Difficult!

Problem 10. Use properties of Fourier transform to evaluate the following integrals. (Do not integrate directly. Recall that $\text{sinc}(x) = \frac{\sin(x)}{x}$.) Clearly state the property or properties that you use.

(a) $\int\limits_{-\infty}^{\infty} \text{sinc}\left(\sqrt{5}x\right) dx$

(b) $\int\limits_{-\infty}^{\infty} e^{-2\pi f \times 2j} 2\text{sinc}\left(2\pi f\right) \left(e^{-2\pi f \times 5j} 2\text{sinc}\left(2\pi f\right)\right)^* df$

(c) $\int\limits_{-\infty}^{\infty} \text{sinc}\left(\sqrt{5}x\right) \text{sinc}\left(\sqrt{7}x\right) dx$

(d) $\int\limits_{-\infty}^{\infty} \text{sinc}\left(\pi\left(x-5\right)\right) \text{sinc}\left(\pi\left(x-\frac{7}{2}\right)\right) dx$