Sirindhorn International Institute of Technology
Thammasat University at Rangsit
School of Information, Computer and Communication Technology

ECS 455: Problem Set 6 Solution

Semester/Year: **2/2010**
Course Title: Mobile Communications
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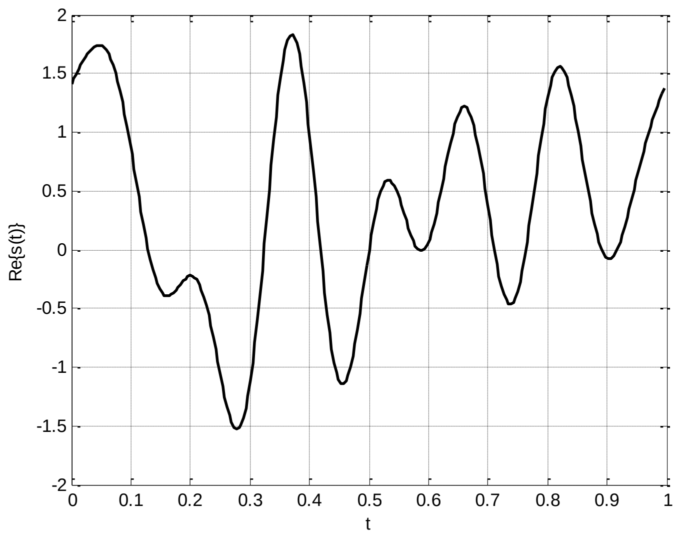
Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

1. **Solution**
   a.



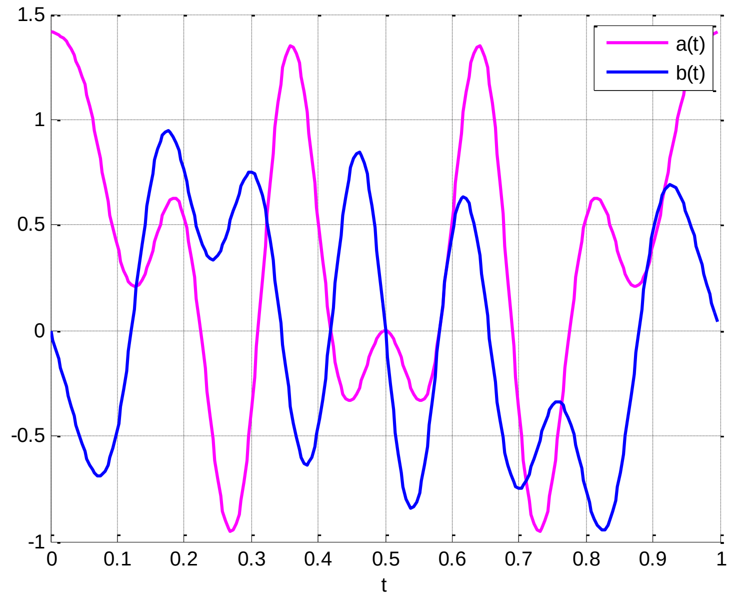
   b. $\mathrm{Re}\{s(t)\} = a(t) - b(t)$
   c. $\mathrm{Re}\{s_2(t)\} = a(t) + b(t)$

d. From part (b) and (c), we have

$$a(t) = \frac{\mathrm{Re}\{s_2(t)\} + \mathrm{Re}\{s(t)\}}{2}$$

and

$$b(t) = \frac{\mathrm{Re}\{s_2(t)\} - \mathrm{Re}\{s(t)\}}{2}.$$



2. Consider the discrete-time complex FIR channel model

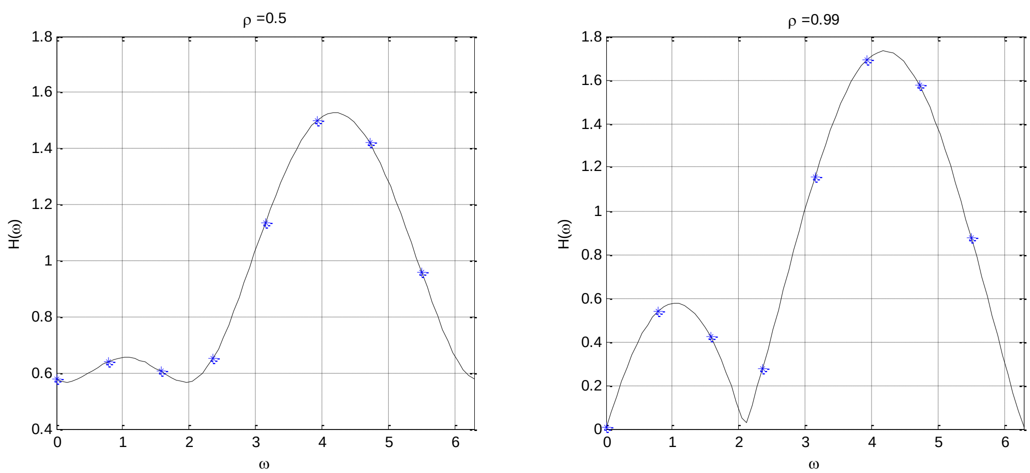
$$y[n] = \{h * x\}[n] + w[n] = \sum_{m=0}^{2} h[m]x[n-m] + w[n]$$

where $w[n]$ is zero-mean additive Gaussian noise.

In this question, assume that *h*[*n*] has unit energy and that $H(z)$ has two zeros at $z_1 = \rho e^{j\frac{2\pi}{3}}$ and $z_2 = \frac{1}{\rho}$ where $\rho < 1$.

Solution

a. The plots of $\left|H(e^{j\omega})\right| = \left|H(z)\right|_{z=e^{j\omega}}\right|$ in the range $0 \le \omega \le 2\pi$ for ρ = 0.5 and 0.99 are shown below:



b. For OFDM system with block size *N* = 8, find the corresponding channel gains $H_k = H(z)\big|_{z=e^{j\frac{2\pi}{N}k}}$, *k* = 0, 1, 2, …, *N*-1 for ρ = 0.5 and 0.99.

Ch # k	$\rho = 0.5$		$\rho = 0.99$	
	$H\left(e^{j\frac{2\pi}{N}k}\right)$	$\left\|H\left(e^{j\frac{2\pi}{N}k}\right)\right\|$	$H\left(e^{j\frac{2\pi}{N}k}\right)$	$\left\|H\left(e^{j\frac{2\pi}{N}k}\right)\right\|$
0	-0.5455 + 0.1890i	0.5774	-0.0087 + 0.0050i	0.0100
1	0.1407 + 0.6246i	0.6403	0.5170 + 0.1489i	0.5380
2	0.4657 + 0.3858i	0.6047	0.3710 - 0.2026i	0.4227
3	0.4649 + 0.4555i	0.6508	-0.0624 + 0.2716i	0.2787
4	0.9820 + 0.5669i	1.1339	0.5860 + 0.9949i	1.1547
5	1.4881 - 0.1882i	1.4999	1.6375 + 0.4284i	1.6927
6	0.8436 - 1.1417i	1.4196	1.3609 - 0.7973i	1.5773
7	-0.3480 - 0.8919i	0.9574	0.2171 - 0.8489i	0.8762

3. All symbol error rates (SER) should be 0 because there is no noise.

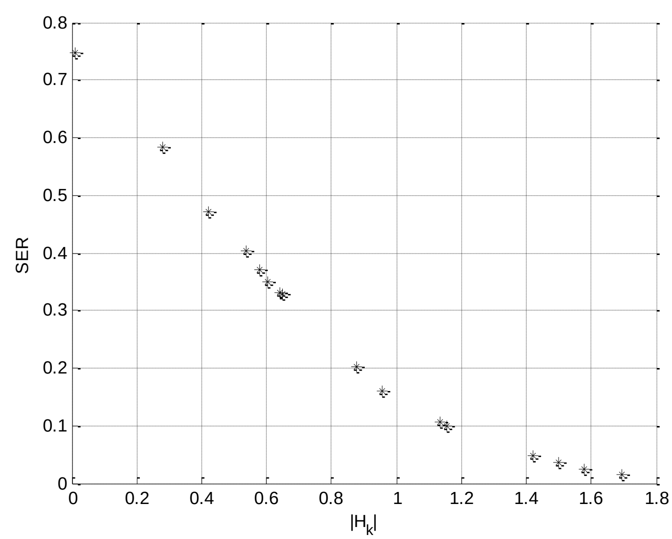
4. In this question, the channel noise is generally non-zero. $w[n]$ is now i.i.d. complex-valued Gaussian noise. Its real and imaginary parts are i.i.d. zero-mean Gaussian with variance N_0 /2 where $N_0 = 1$.

Solution

a.

	$\rho = 0.5$		$\rho = 0.99$	
Ch # k	$\lvert H_k \rvert$	SER	$\lvert H_k \rvert$	SER
0	0.5774	0.3631	0.0100	0.7487
1	0.6403	0.3316	0.5380	0.3919
2	0.6047	0.3601	0.4227	0.4736
3	0.6508	0.3292	0.2787	0.5750
4	1.1339	0.1032	1.1547	0.0973
5	1.4999	0.0328	1.6927	0.0145
6	1.4196	0.0423	1.5773	0.0235
7	0.9574	0.1684	0.8762	0.2028

b. If you try to plot $|H_k|$ vs. SER, you should get something similar to the plot below.



So, $|H_k|$ and SER go in the opposite directions. The channel that has large value of $|H_k|$ will have very good SER performance; that is it will has low SER. Furthermore, the SER of ch#0 when H_0 is about 0 should be very close to 0.75. This is because the channel gain destroys almost all the information contained in the received signal. Hence, the ML detector will be correct with probability 0.5 for each dimension. The complex number (QPSK symbol) has two dimensions. Hence, the chance that it will be decoded correctly is $0.5 \times 0.5 = 0.25$.

If you are very good at digital communications, you may check that the SER is given by

$$2p - p^2 \text{ where } p = Q\left(|H_k| \sqrt{\frac{2}{N_0}} \right).$$