Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 1

Semester/Year: **1/2010**
Course Title: Mobile Communications
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Due date: Nov 23, 2010 (Tuesday)

Instructions

1. ONE sub-question will be graded. Of course, you do not know which part will be selected; so you should work on all of them.
2. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Please submit your solutions for the following questions.

1. In class, we have seen how to use the Euler's formula to show that $\cos^2 x = \frac{1}{2}\left(\cos(2x)+1\right)$.

   For this question, use similar technique to show that

$$\cos A \cos B = \frac{1}{2}\left(\cos(A+B)+\cos(A-B)\right).$$

2. Plot the signal $x(t)$ whose Fourier transform is given by

$$X(f) = \text{sinc}^2(5\pi f) = \left(\frac{\sin(5\pi f)}{(5\pi f)}\right)^2.$$

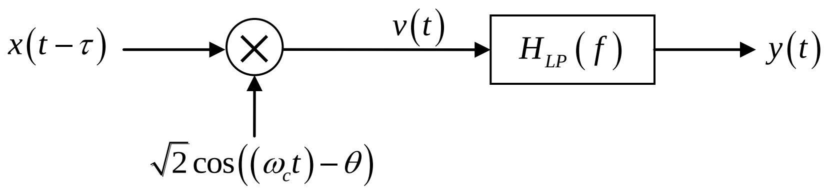
3. Consider the basic DSB-SC transceiver presented in class. Recall that the input of the receiver is

$$x(t-\tau) = m(t-\tau)\sqrt{2}\cos\left(\omega_c(t-\tau)\right)$$

where $m(t) \xrightarrow{\mathcal{F}} M(f)$ is bandlimited to *W*, i.e., $|M(f)| = 0$ for $|f| > W$. We also assume where $f_c \gg W$.

a. Suppose that, at the receiver, we multiply by $\sqrt{2}\cos((\omega_c t) - \theta)$ instead of $\sqrt{2}\cos(\omega_c t)$.

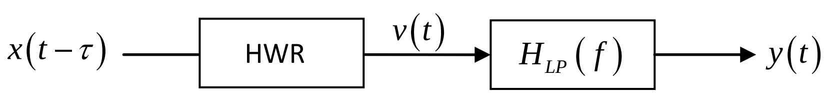
   The new receiver is illustrated below:



   Assume $H_{LP}(f) = \begin{cases} 1, & |f| \le W \\ 0, & \text{otherwise.} \end{cases}$

   Find $y(t)$ (the output of the LPF).

b. Use the same assumptions as part (a). However, at the receiver, instead of multiplying by $\sqrt{2}\cos((\omega_c t) - \theta)$, we pass $x(t-\tau)$ through a half-wave rectifier (HWR).



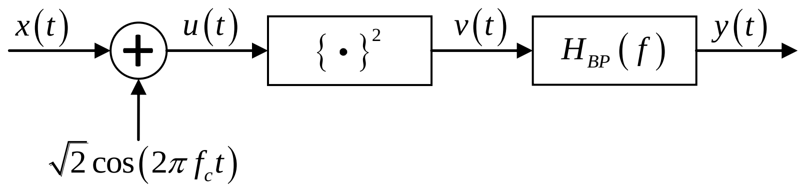
   Make an extra assumption that $m(t) \ge 0$ for all time *t* and that the half-wave rectifier input-output relation is described by a function $f(\bullet)$:

$$f(x) = \begin{cases} x, & x \ge 0, \\ 0, & x < 0. \end{cases}$$

   Find $y(t)$ (the output of the LPF).

4. This question further explores DSB-SC.

   a. Let $x(t) = A_c m(t)$ where $m(t) \xrightarrow{\mathcal{F}} M(f)$ is bandlimited to *W*, i.e., $|M(f)| = 0$ for $|f| > W$. Consider the following block diagram:



where $f_c \gg W$ and

$$H_{BP}(f) = \begin{cases} 1, & |f - f_c| \le W \\ 1, & |f + f_c| \le W \\ 0, & \text{otherwise.} \end{cases}$$

The block labeled "$\{\bullet\}^2$" has output $v(t)$ that is the square of the input $u(t)$:

$$v(t) = u^2(t).$$

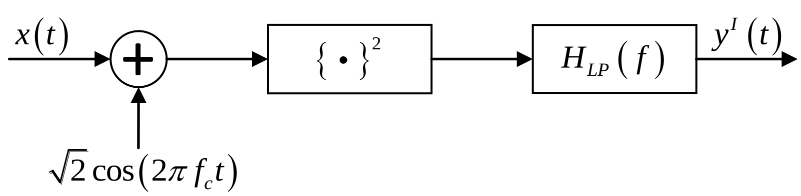
Give a simplified formula for $y(t)$.

b. The block diagram in part (a) is a nice implementation of a modulator because it may be easier to build a squarer than to build a multiplier. Based on the successful use of a squaring operation in the modulator, we decide to use the same squaring operation in the demodulator. Let

$$x(t) = A_c m(t)\sqrt{2}\cos(2\pi f_c t)$$

where $m(t)$ is bandlimited to W, i.e. $|M(f)| = 0$ for $|f| > W$, and $f_c \gg W$.

Consider the following block diagram:



where

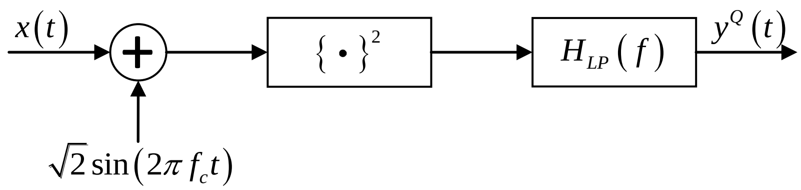
$$H_{LP}(f) = \begin{cases} 1, & |f| \le W \\ 0, & \text{otherwise.} \end{cases}$$

Give a formula for $y^I(t)$. Does this block diagram work as a demodulator; that is, is $y^I(t)$ proportional to $m(t)$?

c. Due to the failure in part (b), we have to think hard and it seems natural to consider also the block diagram with cos replaced by sin. Let $x(t) = A_c m(t)\sqrt{2}\cos(2\pi f_c t)$

where $m(t)$ is bandlimited to *W*, i.e. $|M(f)| = 0$ for $|f| > W$, and $f_c \gg W$ as in part (b).

Consider the following block diagram:



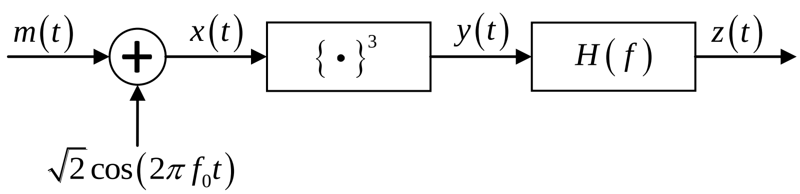
where

$$H_{LP}(f) = \begin{cases} 1, & |f| \le W \\ 0, & \text{otherwise.} \end{cases}$$

Give a formula for $y^Q(t)$.

d. Combining the results of parts (b) and (c), draw a block diagram of a successful DSB-SC demodulator using squaring operations instead of multipliers.

5. Consider the following block diagram:



where $\{\bullet\}^3$ indicates a device whose output is the cube of its input. Let $m(t) \xrightarrow{\mathcal{F}} M(f)$ be band-limited to *W*, that is, $|M(f)| = 0$ for $|f| > W$.

a. Plot an $H(f)$ such that $z(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$. What is the gain in $H(f)$? What is the value of f_c? Notice that the frequency of the cosine is f_0 not f_c. You are supposed to determine f_c in terms of f_0.

b. Let $M(f)$ be

$$M(f) = \begin{cases} 1, & |f| \le W \\ 0, & \text{otherwise.} \end{cases}$$

i. Plot $X(f)$.

ii. Plot $Y(f)$. Hint:

$$M(f) * M(f) = \begin{cases} 2W - |f|, & |f| \le 2W \\ 0, & \text{otherwise.} \end{cases}$$

Do not make an accurate plot or calculation for the Fourier transform of $m^3(t)$.

iii. For your filter of part (a), plot $z(t)$.