HW1 Sol Q1-2

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2:54 PM

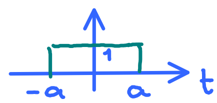
1

$$\cos A \cos B = \left(e^{jA} + e^{-jA}\right)\left(e^{jB} + e^{-jB}\right) \times \frac{1}{4}$$

$$= \Big(\underbrace{e^{j(A+B)} + e^{-j(A+B)}}_{2\cos(A+B)} + \underbrace{e^{j(A-B)} + e^{-j(A-B)}}_{2\cos(A-B)}\Big)\frac{1}{4}$$

$$= \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

2 We know that



$$2a\,\mathrm{sinc}(2\pi a f) \xrightarrow{\mathcal{F}^{-1}} 1[|t| \le a]$$ ← shown in class.

Therefore,

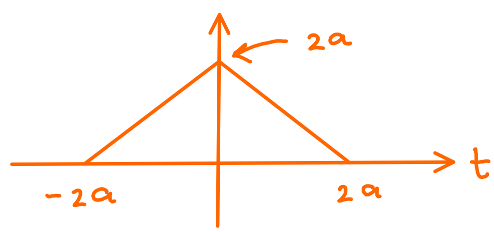
$$\mathrm{sinc}(2\pi a f) \xrightarrow{\mathcal{F}^{-1}} \frac{1}{2a} 1[|t| \le a].$$

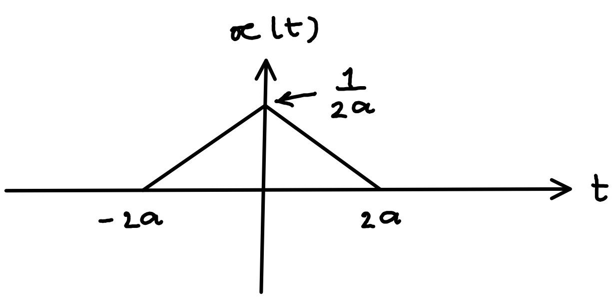
Finally,

$$\mathrm{sinc}^2(2\pi a f) \xrightarrow{\mathcal{F}^{-1}} \frac{1}{2a} 1[|t| \le a] * \frac{1}{2a} 1[|t| \le a]$$

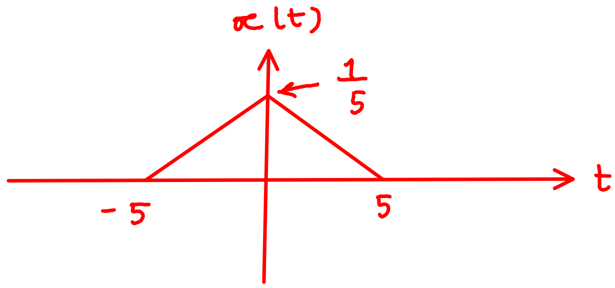
$$= \frac{1}{4a^2}\left(1[|t| \le a] * 1[|t| \le a]\right)$$

So, we can solve this question if we can find the convolution of $1[|t| \le a]$ with itself.





For us, $a = \frac{5}{2}$. So, $2a = 5$ and the plot of $x(t)$ is



HW1 Sol Q3

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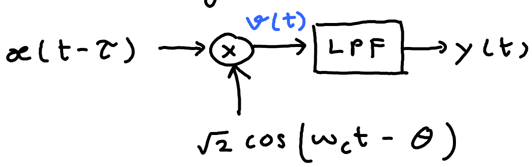
3

3.a

Since the modification is made at the receiver, any results before it is unchanged and therefore we still have



where

$$x(t-\tau) = m(t-\tau)\sqrt{2}\cos(\omega_c t - \underbrace{\omega_c\tau}_{\equiv \phi \text{ as defined in lecture.}})$$

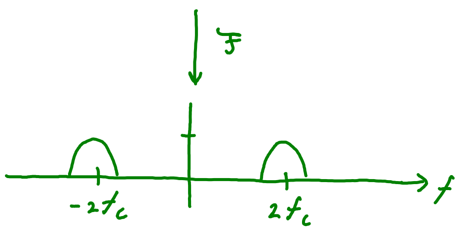
Let $v(t)$ be the signal before the LPF.

Then $v(t) = x(t-\tau) \times \sqrt{2}\cos(\omega_c t - \theta)$

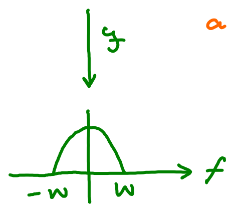
$$= 2\,m(t-\tau)\cos(\omega_c t - \phi)\cos(\omega_c t - \theta)$$

$$= m(t-\tau)\left(\cos(2\omega_c t - \phi - \theta) + \cos(\theta - \phi)\right)$$

$$= \underbrace{m(t-\tau)\cos(2\omega_c t - \phi - \theta)}\ + \underbrace{m(t-\tau)}\ \underbrace{\cos(\theta-\phi)}_{\text{a constant}}$$



This won't pass the LPF.



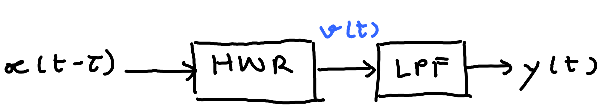
This will pass the LPF.

$$y(t) = m(t-\tau)\cos(\theta - \phi) = m(t-\tau)\cos(\theta - \omega_c\tau).$$

3.b

Again, we have

$$x(t-\tau) = m(t-\tau)\sqrt{2}\cos(\omega_c(t-\tau))$$



Let $v(t)$ be the signal before the LPF.

Then, $v(t) = x(t-\tau) \times \underbrace{w(t-\tau)}$

Because $m(t-\tau)$ is always ≥ 0, the sign of $x(t-\tau)$ only depends on $\cos(\omega_c(t-\tau))$, which is simply a shifted version of $\cos(\omega_c t)$.

All of the analysis is the same as what was presented in class except that we now have a time shift of amount τ.

$$y(t) = \frac{\sqrt{2}}{\pi} m(t-\tau)$$

HW1 Sol Q4-5

Monday, November 23, 2009

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4

(a) $x(t) = A_c m(t) \xrightarrow{\mathcal{F}} X(f) = A_c M(f)$

So, $X(f)$ is also bandlimited to W.

$u(t) = x(t) + \sqrt{2}\cos(\omega_c t)$ $\omega_c = 2\pi f_c$

$v(t) = u^2(t) = \left(x(t) + \sqrt{2}\cos(\omega_c t)\right)^2$

$= x^2(t) + 2\sqrt{2}\,x(t)\cos(\omega_c t) + \underbrace{2\cos^2(\omega_c t)}_{1+\cos(2\omega_c t)}$

$= \underbrace{\left(1 + x^2(t)\right)}_{\xrightarrow{BPF}\ 0} + 2\sqrt{2}\,x(t)\cos\omega_c t + \underbrace{\cos(2\omega_c t)}_{\xrightarrow{BPF}\ 0}$

Note1: $x^2(t) \xrightarrow{\mathcal{F}} X(f) * X(f)$

So, $x^2(t)$ is bandlimited to 2W

Because $f_c \gg W$, the spectrum of $x^2(t)$ will <u>not</u> be in the passband of the BPF which centers around f_c.

Note 2: The term $\cos(2\omega_c t)$ is at frequency $2 \times f_c$ which again is outside the passband.

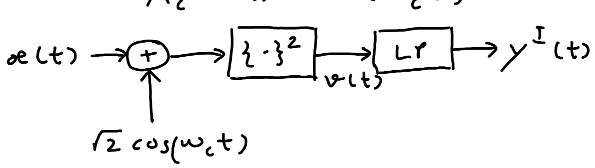
$y(t) = BPF\{v(t)\}$

$= 2\sqrt{2}\,x(t)\cos\omega_c t$

$= \boxed{2\sqrt{2}\,A_c\, m(t)\cos\omega_c t}$

(b) Assume

$x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$



From the above figure,

$v(t) = \left(x(t) + \sqrt{2}\cos(\omega_c t)\right)^2$

$= 2\cos^2(\omega_c t)\left(A_c m(t) + 1\right)^2$

HW1 Sol Q4-5

Monday, November 23, 2009

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4

(a) $x(t) = A_c m(t) \xrightarrow{\mathcal{F}} X(f) = A_c M(f)$

So, $X(f)$ is also bandlimited to W.

$$u(t) = x(t) + \sqrt{2}\cos(\omega_c t) \qquad \omega_c = 2\pi f_c$$

$$v(t) = u^2(t) = (x(t) + \sqrt{2}\cos(\omega_c t))^2$$

$$= x^2(t) + 2\sqrt{2}\,x(t)\cos(\omega_c t) + \underbrace{2\cos^2(\omega_c t)}_{1+\cos(2\omega_c t)}$$

$$= \underbrace{(1 + x^2(t))}_{\text{BPF} \to 0} + 2\sqrt{2}\,x(t)\cos\omega_c t + \underbrace{\cos(2\omega_c t)}_{\text{BPF} \to 0}$$

Note1: $x^2(t) \xrightarrow{\mathcal{F}} X(f) * X(f)$

So, $x^2(t)$ is bandlimited to 2W

Because $f_c \gg W$, the spectrum of $x^2(t)$ will <u>not</u> be in the passband of the BPF which centers around f_c.

Note 2: The term $\cos(2\omega_c t)$ is at frequency $2 * f_c$ which again is outside the passband.

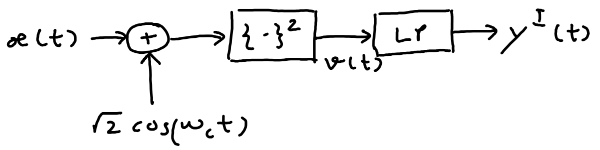
$$y(t) = \text{BPF}\{v(t)\}$$

$$= 2\sqrt{2}\,x(t)\cos\omega_c t$$

$$= \boxed{2\sqrt{2}\,A_c m(t)\cos\omega_c t}$$

(b) Assume

$$x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$$



From the above figure,

$$v(t) = (x(t) + \sqrt{2}\cos(\omega_c t))^2$$

$$= 2\cos^2(\omega_c t)\,(A_c m(t) + 1)^2$$

$$= 1 + \cos(2\omega_c t)\,(A_c^2 \underbrace{m^2(t)}_{\substack{\text{spectrum}\\ \text{is from}\\ [-2W,\,2W]}} + 1 + 2A_c \underbrace{m(t)}_{\substack{\text{spectrum}\\ \text{is from}\\ [-W,\,W]}})$$

$g(t)$

$$= g(t) + \underbrace{g(t)\cos(2\omega_c t)}_{\xrightarrow{LPF}\ 0}$$

Note 1: We know that $g(t)$ is band limited to $[-2W, 2W]$ because all of its terms are band limited to $[-2W, 2W]$. So, only some parts of it will pass through the LPF.

Note 2: $g(t)\cos(2\omega_c t)$ is centered @ $2f_c$ and therefore will not pass thought the LPF.

$$y^I(t) = LPF\{v(t)\}$$
$$= LPF\{g(t)\}$$
$$= \boxed{1 + 2A_c m(t) + LPF\{A_c^2 m^2(t)\}}$$

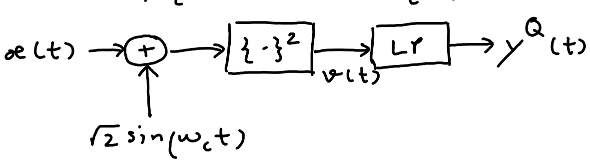
This term has spectrum beyond 2W. So, only a portion of it will pass through the LPF.

$y^I(t)$ is $\boxed{\text{not}}$ proportional to $m(t)$.

Hence, this block diagram does not work as a demodulator.

(c) Assume

$x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$ as in part (b).



We then have

$$v(t) = \left(x(t) + \sqrt{2}\sin(\omega_c t)\right)^2$$
$$= 2\left(A_c m(t)\cos(\omega_c t) + \sin(\omega_c t)\right)^2$$
$$= 2\left(A_c^2 m^2(t)\cos^2(\omega_c t) + A_c m(t)\cos(\omega_c t)\sin(\omega_c t) + \sin^2(\omega_c t)\right)$$
$$= 2\left(A_c^2 m^2(t)\cos^2(\omega_c t) + \sin^2(\omega_c t)\right) + A_c m(t)\sin(2\omega_c t)$$
$$= 2\left(\left(A_c^2 m^2(t) - 1\right)\cos^2(\omega_c t) + 1\right) + A_c m(t)\sin(2\omega_c t)$$
$$= 2 + \left(A_c^2 m^2(t) - 1\right)\left(1 + \underbrace{\cos(2\omega_c t)}_{\xrightarrow{LPF}\ 0}\right) + \underbrace{A_c m(t)\sin(2\omega_c t)}_{\xrightarrow{LPF}\ 0}$$

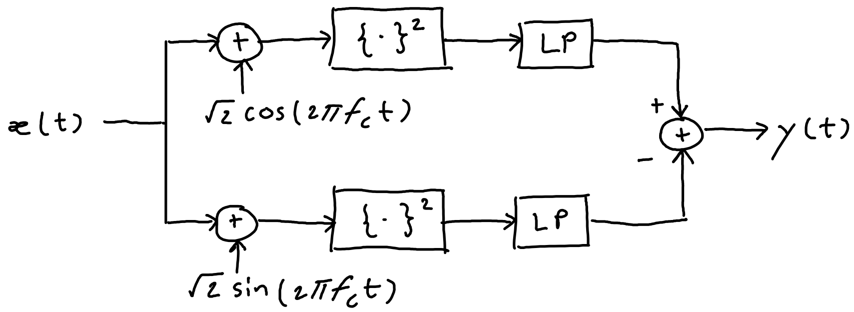
$$y^Q(t) = 2 + LPF\{A_c^2 m^2(t)\} - 1$$
$$= \boxed{LPF\{A_c^2 m^2(t)\} + 1}$$

$$= \boxed{LPF\left\{A_c^2 m^2(t)\right\} + 1}$$

(d) Observe that

$$\underbrace{y^I(t)}_{\text{from (b)}} - \underbrace{y^Q(t)}_{\text{from (c)}} = 2A_c m(t)$$ which is the desired output of a successful DSB-SC demodulator.

Hence, the following block diagram would work:



5

(a) $y(t) = \left(m(t) + \sqrt{2}\cos(2\pi f_0 t)\right)^3$

$$= m^3(t) + 3m^2(t)\sqrt{2}\cos\omega_0 t + \underbrace{3m(t)\,2\cos^2\omega_0 t}_{\begin{subarray}{l}= 3m(t)(1+\cos 2\omega_0 t)\\ = 3m(t) + 3m(t)\cos(2\omega_0 t)\end{subarray}} + \underbrace{(\sqrt{2})^3\cos^3(\omega_0 t)}_{= \frac{3}{\sqrt{2}}\cos(\omega_0 t) + \frac{1}{\sqrt{2}}\cos(3\omega_0 t)}$$

$$\left\{\begin{aligned} 2\cos^2(\theta) &= 1 + \cos(2\theta) \\ 2\cos^3(\theta) &= \cos\theta + \cos\theta\cos 2\theta \\ &= \cos\theta + \tfrac{1}{2}\cos\theta + \tfrac{1}{2}\cos 3\theta \\ &= \tfrac{3}{2}\cos\theta + \tfrac{1}{2}\cos 3\theta \end{aligned}\right\}$$

We want $z(t) = m(t)\sqrt{2}\cos(\omega_c t)$.

We see that the only term in $y(t)$ that has the form

constant $\times$ m $\times$ $\cos(\ )$

is $3m(t)\cos(2\omega_0 t)$.

Therefore, we will center the passband to cover this part and adjust the gain to make the output the same as $z(t)$.

In particular,

We need to make $\boxed{\begin{aligned} 2f_0 &= f_c. \\ \text{So, } f_0 &= f_c/2. \end{aligned}}$

Let $H_{BP}(f) = \begin{cases} c, & |f - f_c| \le W \\ c, & |f + f_c| \le W \\ 0, & \text{otherwise} \end{cases}$

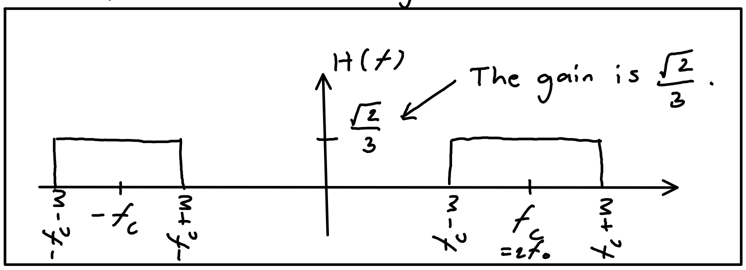
Then,

$$z(t) = \underbrace{C \times 3}_{\downarrow}\, m(t)\cos(2\omega_0 t)$$

We need $C \times 3 = \sqrt{2}$

$$C = \frac{\sqrt{2}}{3}$$

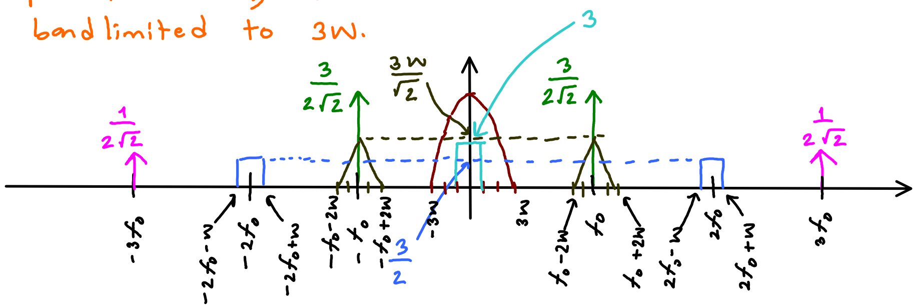
The plot of $H(f)$ is given below:



(b) From (a), we have

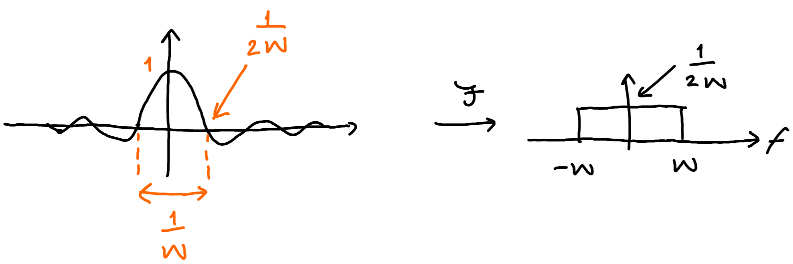
$$y(t) = m^3(t) + 3m(t) + 3\sqrt{2}\, m^2(t)\cos(\omega_0 t) + \frac{3}{\sqrt{2}}\cos(\omega_0 t) + 3m(t)\cos(2\omega_0 t) + \frac{1}{\sqrt{2}}\cos(3\omega_0 t)$$

Without trying to make and accurate plot for $m^3(t)$, we know that it is band limited to 3W.

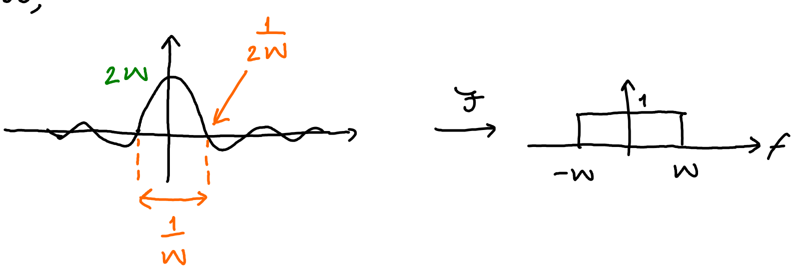


(c) $z(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$

We know that



So,



$z(t)$ is the above sinc. function multiplied

$z(t)$ is the above sinc. function multiplied by $\sqrt{2}\cos(2\pi f_c t)$.

Because $f_c \gg W$, we know that

$$\frac{1}{W} \gg \frac{1}{f_c}$$

↑ period of cos.

So, the sinc function becomes the envelope of the cosine carrier.