

25: Cyclic Prefix

Tuesday, February 15, 2011
1:06 PM

Lecture 25

Announcement : Submit HW5 now.

HW6 is posted.

Due on Friday, Feb 25

(Last day of class)

Recall: DFT and FFT

sequence (in discrete time) $x[n]$

$$z\text{-transform} : X(z) = \sum_n x[n] z^{-n}$$

$$\text{DTFT} : X(e^{j\omega}) = \sum_n x[n] e^{-j\omega n}$$

(Discrete time
Fourier transform)

DFT : $x[n]$ is N -point ($n=0, 1, 2, \dots, N-1$)

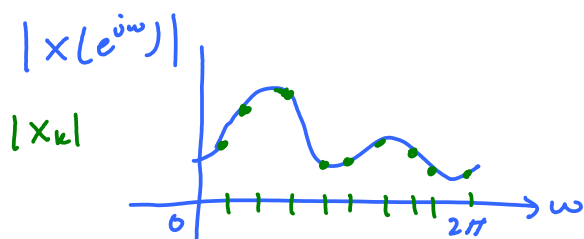
$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x[n] e^{-j\omega n} \quad (\text{DTFT}) \quad 0 \leq \omega < 2\pi$$

$$X_k = \sum_{n=0}^{N-1} x[n] e^{-jn \left(\frac{2\pi}{N} k \right)} = \sum_{n=0}^{N-1} x[n] e^{-jn \omega_k} \quad (\text{DFT})$$

$$\omega_k = k \frac{2\pi}{N} \quad 0 \leq k < N$$

$$X_k = X(e^{j\omega}) \Big|_{\omega = \omega_k = k \frac{2\pi}{N}}$$

$$\omega_k = 0, \frac{2\pi}{N}, 2 \frac{2\pi}{N}, 3 \frac{2\pi}{N}, \dots, (N-1) \frac{2\pi}{N}$$



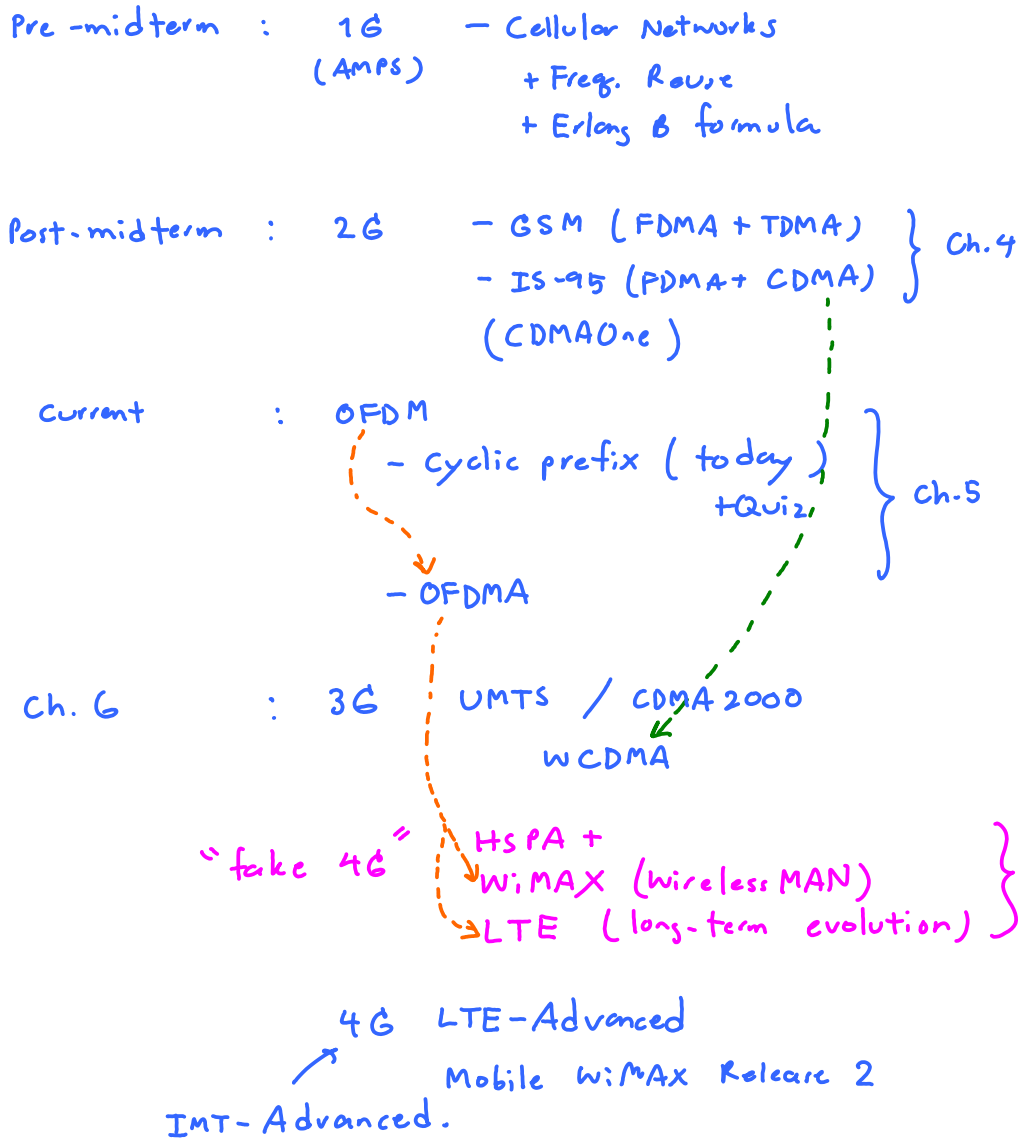
conclusion DFT is a sampled version of DTFT.

(FFT)

Overview of our class

Multiple-Access Techniques in Mobile Communications.

main idea : orthogonality



Circular Convolution

Example : Convolution

$$[1 \ 2 \ 3] * [4 \ 5 \ 6] = [4 \ 13 \ 28 \ 27 \ 18]$$

$$\begin{array}{r}
 1 \ 2 \ 3 \\
 6 \ 5 \ 4 \\
 6 \ 5 \ 4
 \end{array}$$

$$1 \times 4 = 4$$

$$5 \times 1 + 2 \times 4 = 5 + 8 = 13$$

$$\begin{array}{r}
 6 \ 5 \ 4 \quad 1 \times 6 + 2 \times 5 + 3 \times 4 = 6 + 10 + 12 = 28 \\
 \quad 6 \ 5 \ 4 \quad 6 \times 2 + 5 \times 3 = 12 + 15 = 27 \\
 \quad \quad 6 \ 5 \ 4 \quad 6 \times 3 = 18
 \end{array}$$

Example : Circular Convolution

$$[1 \ 2 \ 3] \circledast [4 \ 5 \ 6]$$

$$\begin{array}{r}
 1 \ 2 \ 3 \ 1 \ 2 \ 3 \ 1 \ 2 \ 3 \ 1 \ 2 \ 3 \\
 \quad 6 \ 5 \ 4 \quad 2 \times 6 + 3 \times 5 + 1 \times 4 = 12 + 15 + 4 = 31 \\
 \quad \quad 6 \ 5 \ 4 \quad 6 \times 3 + 1 \times 5 + 2 \times 4 = 18 + 5 + 8 = 31 \\
 \quad \quad \quad 6 \ 5 \ 4 \quad 1 \times 6 + 2 \times 5 + 3 \times 4 = 6 + 10 + 12 = 28
 \end{array}$$

Conclusion :

circular convolution is an operation performed between two vectors of the same size (N)

To do it, proceed as in usual convolution

- except 1) need to replicate one of the vectors
- 2) need to stop when you have N answers (elements).

Example :

$$[1 \ 2 \ 3 \ 0 \ 0] \circledast [4 \ 5 \ 6 \ 0 \ 0]$$

$$\begin{array}{r}
 1 \ 2 \ 3 \ 0 \ 0 \ 1 \ 2 \ 3 \ 0 \ 0 \ 1 \ 2 \ 3 \ 0 \ 0 \\
 \quad 0 \ 0 \ 6 \ 5 \ 4 \\
 \quad \quad 0 \ 0 \ 6 \ 5 \ 4 \\
 \quad \quad \quad 0 \ 0 \ 6 \ 5 \ 4 \\
 \quad \quad \quad \quad 0 \ 0 \ 6 \ 5 \ 4 \\
 \quad \quad \quad \quad \quad 0 \ 0 \ 6 \ 5 \ 4
 \end{array}$$

$$\begin{array}{l}
 1 \times 4 = 4 \\
 5 \times 1 + 4 \times 2 = \dots \\
 6 \times 3 = 18
 \end{array}$$

exactly the same as $[1 \ 2 \ 3] \ast [4 \ 5 \ 6]$ above