ECS455: Chapter 4

Multiple Access

Multiple access

4.6 SSMA and CDMA

Multiple users

Office Hours:
BKD 3601-7
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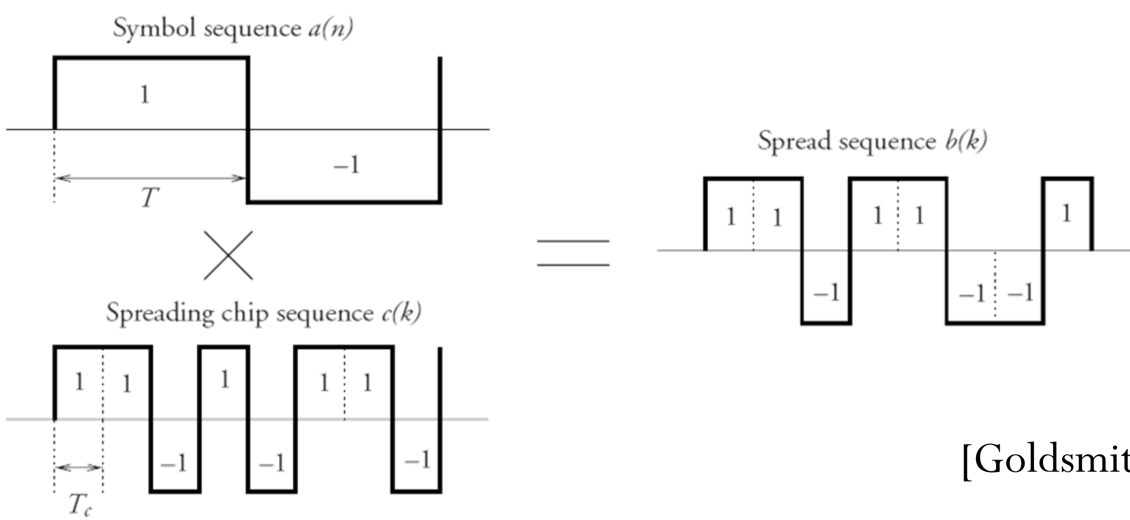
DSSS and m-sequences

- m-sequences
  - **Excellent auto-correlation** properties (for ISI rejection)
  - Highly suboptimal for exploiting the ***multiuser*** capabilities of spread spectrum.
- There are only a small number of maximal length codes of a given length.
- Moreover, maximal length codes generally have relatively **poor cross-correlation** properties, at least for some sets of codes.



[Goldsmith, 2005, Ch 13]

Number of primitive polynomials

Number of different primitive polynomials which can be used to generate m-sequences,

- r is the degree of the primitive polynomials and
- Np is the number of different primitive polynomials available.

r	N_p	r	N_p
2	1	11	176
3	2	12	144
4	2	13	630
5	6	14	756
6	6	15	1800
7	18	16	2048
8	16	17	7710
9	48	18	8064
10	60	19	27594

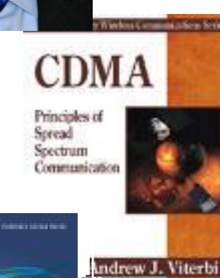
[Chen, 2007, p 145]

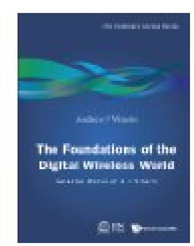
SSMA

- For spread spectrum systems with **multiple users**, codes such as Gold, Kasami, or Walsh codes are used instead of maximal length codes
- Superior cross-correlation properties.
- Worse auto-correlation than maximal length codes.
  - The autocorrelation function of the spreading code determines its multipath rejection properties.

Qualcomm

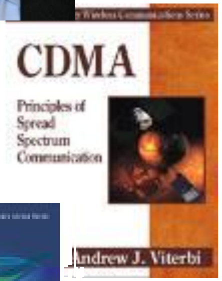
- Qualcomm
- Founders: Two of the most eminent engineers in the world of mobile radio
- Irwin Jacobs is the chairman and founder
  - Cornell (undergrad.: Hotel > EE)
  - MIT (grad.)
  - UCSD (Prof.)
- Andrew J. Viterbi is the co-founder
  - MIT (BS, MS)
  - USC (PhD)
  - UCLA and UCSD (Prof.)
  - Same person that invented the Viterbi algorithm for decoding convolutionally encoded data.

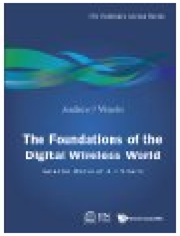




Code Division Multiple Access (CDMA)

- 1991: Qualcomm announced
  - that it had invented a new cellular system based on CDMA
  - that the capacity of this system was 20 or so times greater than any other cellular system in existence
- However, not all of the world was particularly pleased by this apparent breakthrough—in particular, GSM manufacturers became concerned that they would start to lose market share to this new system.
  - The result was continual and vociferous argument between Qualcomm and the GSM manufacturers.





DS-CDMA

- One way to achieve SSMA
- May utilize Direct Sequence Spread Spectrum (DS/SS)
  - Direct sequence is not the only spread-spectrum signaling format suitable for CDMA
- All users use the same carrier frequency and may transmit simultaneously.
- Users are assigned different "**signature waveforms**" or "code" or "codeword" or "**spreading signal**"

Not to be confused with error-correcting codes that add redundancy to combat channel noise and distortion

- The narrowband message signal is multiplied (modulated) by the **spreading signal** which has a very large bandwidth (orders of magnitudes greater than the data rate of the message).
- Each user's codeword is ***approximately orthogonal*** to all other codewords.
- Should not be confused with the mobile phone standards called cdmaOne (Qualcomm's IS-95) and CDMA2000 (Qualcomm's IS-2000) (which are often referred to as simply "CDMA")
  - These standards use CDMA as an underlying channel access method.

Inner Product (Cross Correlation)

- Vector

column vector $\vec{x}, \vec{y}$

row vector

$\underline{x} = (x_1, x_2, \ldots, x_n)$

$\underline{y} = (y_1, y_2, \ldots, y_n)$

Complex conjugate

$$\langle \vec{x}, \vec{y} \rangle = \vec{x} \cdot \vec{y}^* = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}^* = \sum_{k=1}^{n} x_k y_k^*$$

In MATLAB,

1. sum(x.* conj(y))
2. If you have column vectors x, y
   y' * x
3. If you have row vectors x, y
   x * y'

- Waveform: Time-Domain

$$\langle x, y \rangle = \int_{-\infty}^{\infty} x(t) y^*(t) dt$$

- Waveform: Frequency Domain

$$\langle X, Y \rangle = \int_{-\infty}^{\infty} X(f) Y^*(f) df$$

Orthogonality

- Two signals are said to be **orthogonal** if their **inner product** is zero.
- The symbol $\perp$ is used to denote orthogonality.

Vector:

$$\langle \vec{a},\vec{b}\rangle = \vec{a}\cdot\vec{b}^* = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}\cdot\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}^* = \sum_{k=1}^{n} a_k b_k^* = 0$$

Time-domain:

$$\langle a,b\rangle = \int_{-\infty}^{\infty} a(t)\,b^*(t)\,dt = 0$$

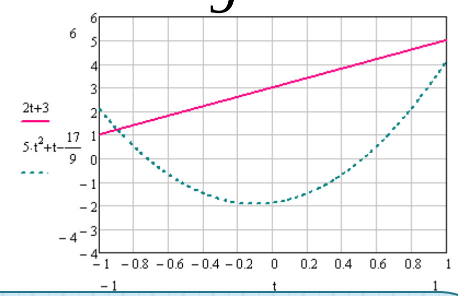
Frequency domain:

$$\langle A,B\rangle = \int_{-\infty}^{\infty} A(f)\,B^*(f)\,df = 0$$

Example:

$$2t+3 \text{ and } 5t^2+t-\frac{17}{9} \text{ on } [-1,1]$$

$$\int_{-1}^{1}(2t+3)\left(5t^2+t-\frac{17}{9}\right)dt = 0$$



Example (Fourier Series):

$$\sin\left(2\pi k_1 \frac{t}{T}\right) \text{ and } \cos\left(2\pi k_2 \frac{t}{T}\right) \text{ on } [0,T]$$

$$e^{j2\pi n\frac{t}{T}} \text{ on } [0,T]$$

Important Properties

- Parseval's theorem

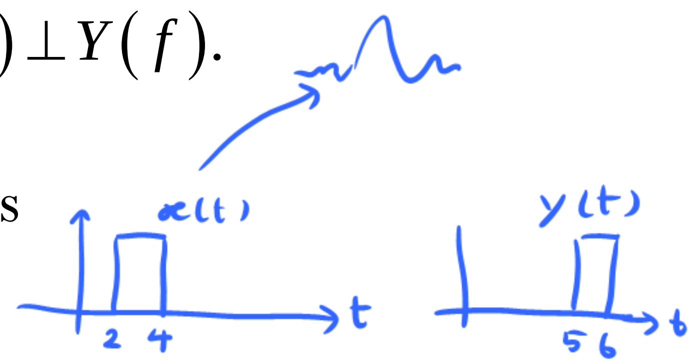
$$\langle x, y\rangle \equiv \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{\infty} X(f)\, Y^*(f)\, df \equiv \langle X, Y\rangle$$

Energy of $x(t)$

1) $\displaystyle\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$

2) If $x(t) \perp y(t)$, then $X(f) \perp Y(f)$.

- If the non-zero regions of two signals
  - do not overlap in time domain or
  - do not overlap in freqeuncy domain,



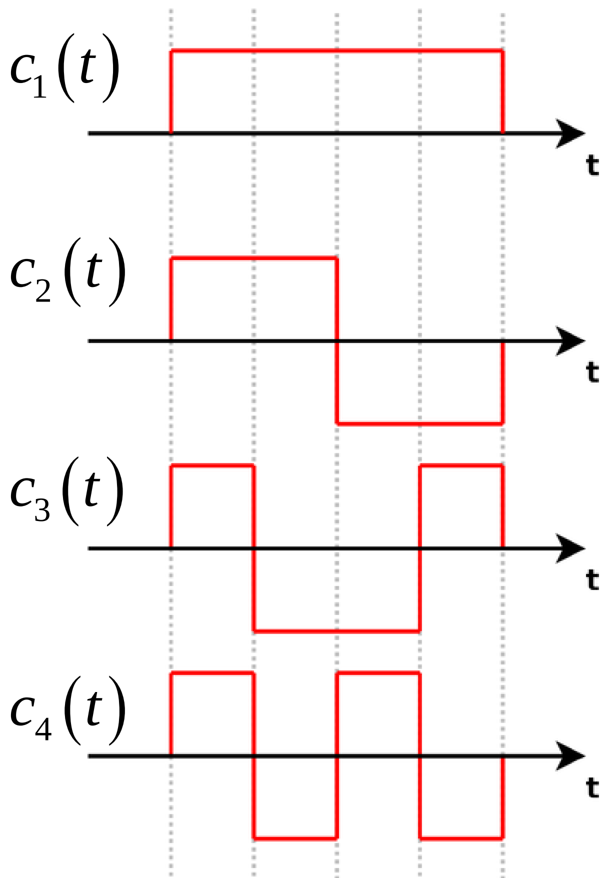
Then the two signals are orthogonal (their inner product = 0).

CDMA

- ***Orthogonal*** signaling > no inter-channel interference
- Special cases:
  - TDMA: The waveforms do not overlap in the time domain.
  - FDMA: The waveforms do not overlap in the frequency domain.
- Orthogonal signals may overlap both in time and in frequency domain.

Example: Orthogonality

An example of four "(mutually) orthogonal" digital signals.



When $i \neq j$,

$$\langle c_i(t), c_j(t) \rangle = 0$$

① Notice that $\int_{-\infty}^{\infty} c_i(t)\,dt = 0$ $\quad i = 2, 3, 4$

② $\langle c_1, c_i \rangle = \int_{-\infty}^{\infty} c_1(t)\, c_i(t)\,dt = 0$ $\quad i = 2, 3, 4$

③ $\int c_2(t) \times c_3(t) = \int c_4(t) = 0$

$\int c_2(t) \times c_4(t) = \int c_3 = 0$

$\int c_3 \times c_4 = \int c_2 = 0$

Orthogonality in Communication

CDMA

$$s(t)=\sum_{k=0}^{\ell-1} S_k c_k(t) \xrightarrow{\mathcal{F}} S(f)=\sum_{k=0}^{\ell-1} S_k C_k(f) \qquad \text{where} \quad c_{k_1} \perp c_{k_2}$$

TDMA

$$s(t)=\sum_{k=0}^{\ell-1} S_k c(t-kT_s) \xrightarrow{\mathcal{F}} S(f)=C(f)\sum_{k=0}^{\ell-1} S_k e^{-j2\pi fkT_s}$$

where $c(t)$ is time-limited to $[0,T]$.

This is a special case of CDMA with $c_k(t)=c(t-kT_s)$

The c_k are non-overlapping in time domain.

FDMA

$$S(f)=\sum_{k=0}^{\ell-1} S_k C(f-k\Delta f)$$

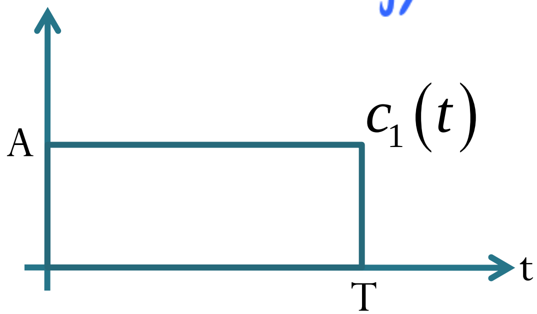
where $C(f)$ is frequency-limited to $[0,\Delta f]$.

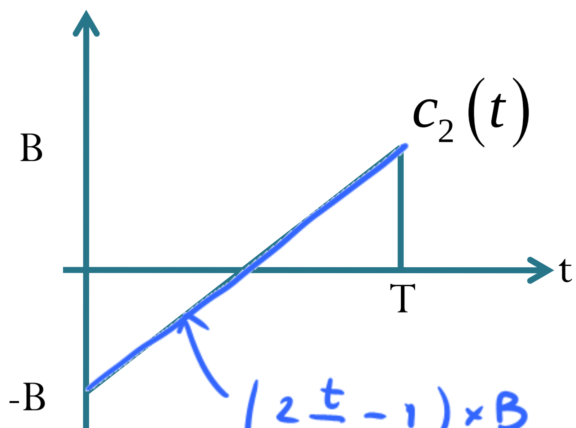
This is a special case of CDMA with $C_k(f)=C(f-k\Delta f)$

The C_k are non-overlapping in freq. domain.

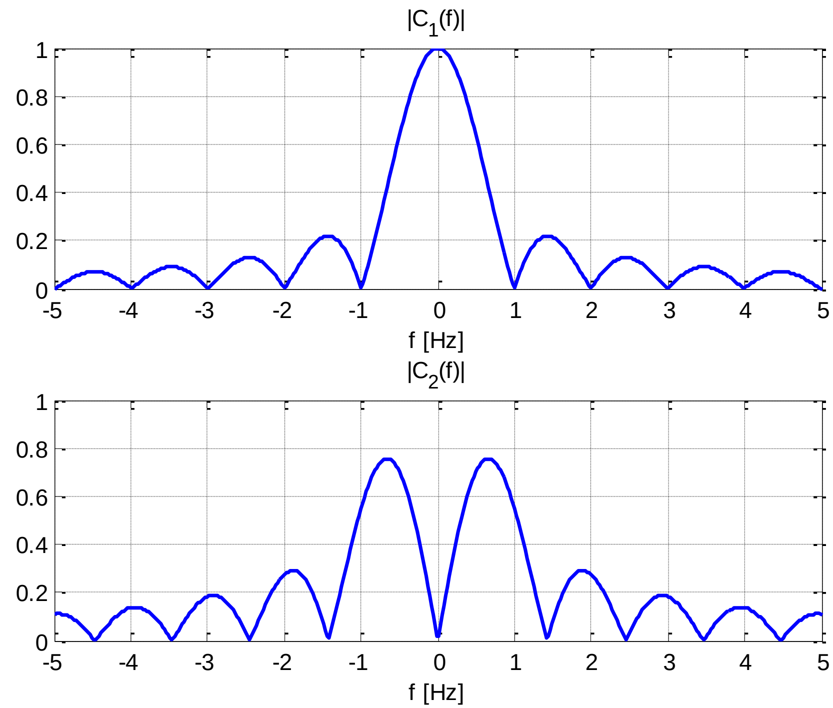
Example 1

Energy of $c_1(t) = \int_{-\infty}^{\infty} |c_1(t)|^2 dt = A^2 T$





$\left(2\frac{t}{T} - 1\right) \times B$

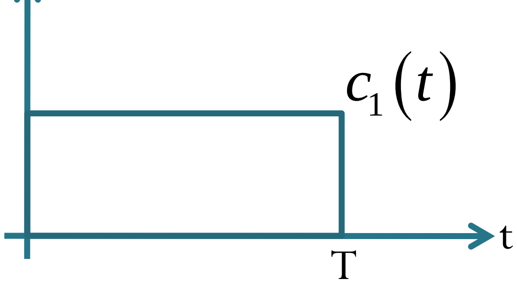


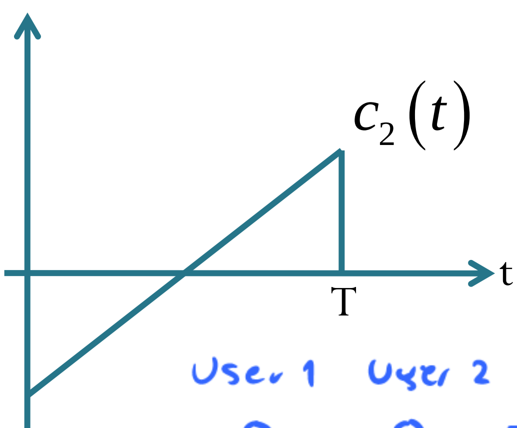
Energy of $c_2(t) = \frac{1}{3} B^2 T$

The two waveforms above overlaps both in time domain and in frequency domian.

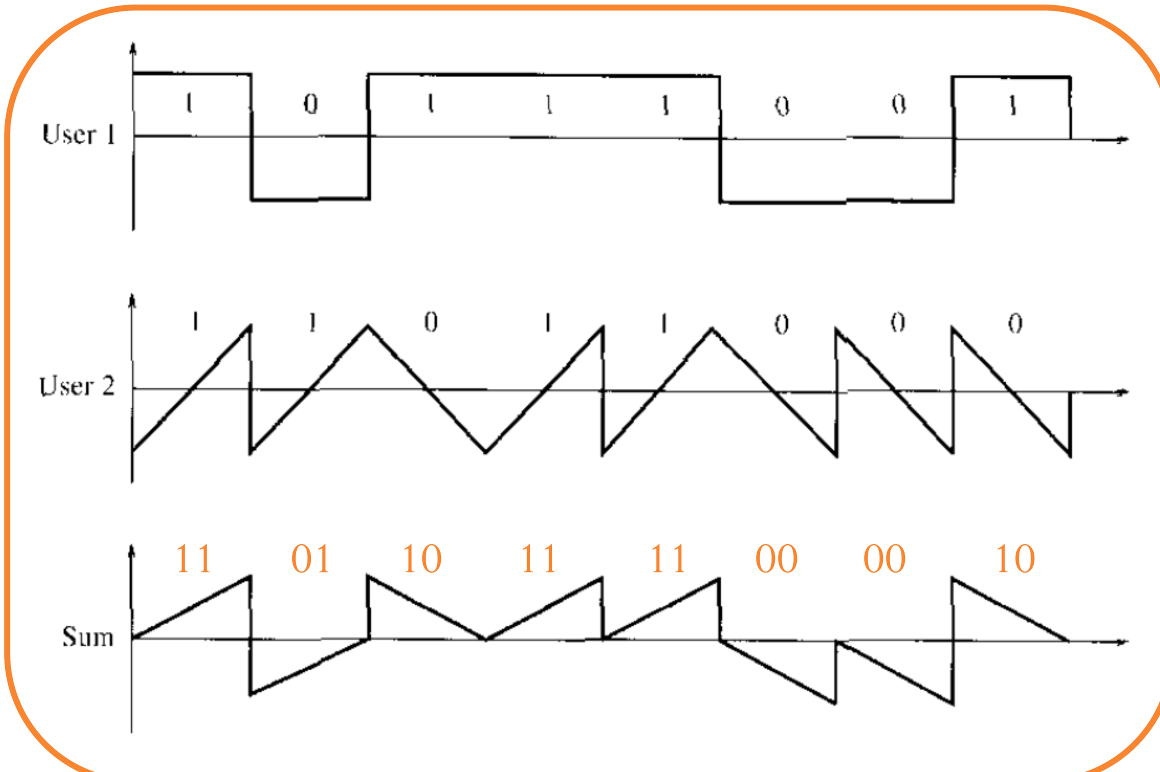
Example 1 (Con't)

Here, we use A = B. It is easy to decode the original waveforms from the shape of the sum.





User 1	User 2
0	0
0	1
1	0
1	1

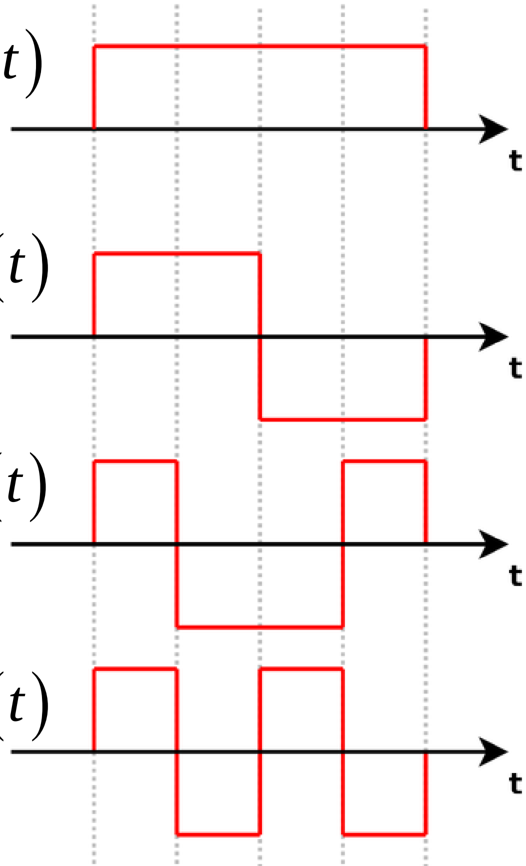


[Figure 1.6, Verdu, 1998]

Need elegant decoding

Example 2: DS-CDMA

$c_1(t)$, $c_2(t)$, $c_3(t)$, $c_4(t)$



Digital version

$$\vec{c}_1 = \begin{bmatrix} +1 & +1 & +1 & +1 \end{bmatrix}$$

$$\vec{c}_2 = \begin{bmatrix} +1 & +1 & -1 & -1 \end{bmatrix}$$

$$\vec{c}_3 = \begin{bmatrix} +1 & -1 & -1 & +1 \end{bmatrix}$$

$$\vec{c}_{\cancel{3}4} = \begin{bmatrix} +1 & -1 & +1 & -1 \end{bmatrix}$$

$$s = \sum_{k=0}^{\ell-1} S_k \vec{c}_k$$

message of user k

$$s(t) = \sum_{k=0}^{\ell-1} S_k c_k(t)$$