December 16, 2010

ECS455: Chapter 3

Poisson process and Markov chain

3.2 Markov Chain

Office Hours:
BKD 3601-7
Tuesday 9:30-10:30
Friday 14:00-16:00

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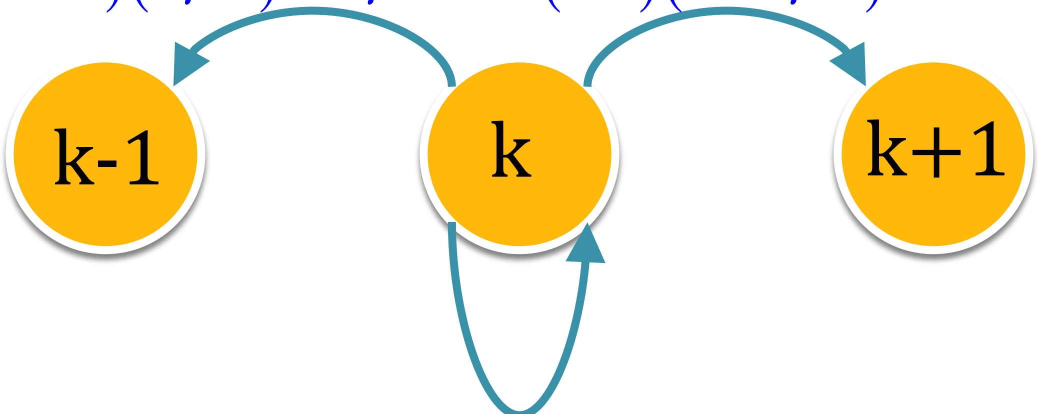
Small slot Analysis (Transition Prob.)

$$K_{i+1} = K_i + (\#\text{ new call request}) - (\#\text{ old-call end})$$

$$(1-\lambda\delta)(k\mu\delta) \approx k\mu\delta \qquad (\lambda\delta)(1-k\mu\delta) \approx \lambda\delta$$



$$(1-\lambda\delta)(1-k\mu\delta)+(\lambda\delta)(k\mu\delta) \approx 1-\lambda\delta-k\mu\delta$$

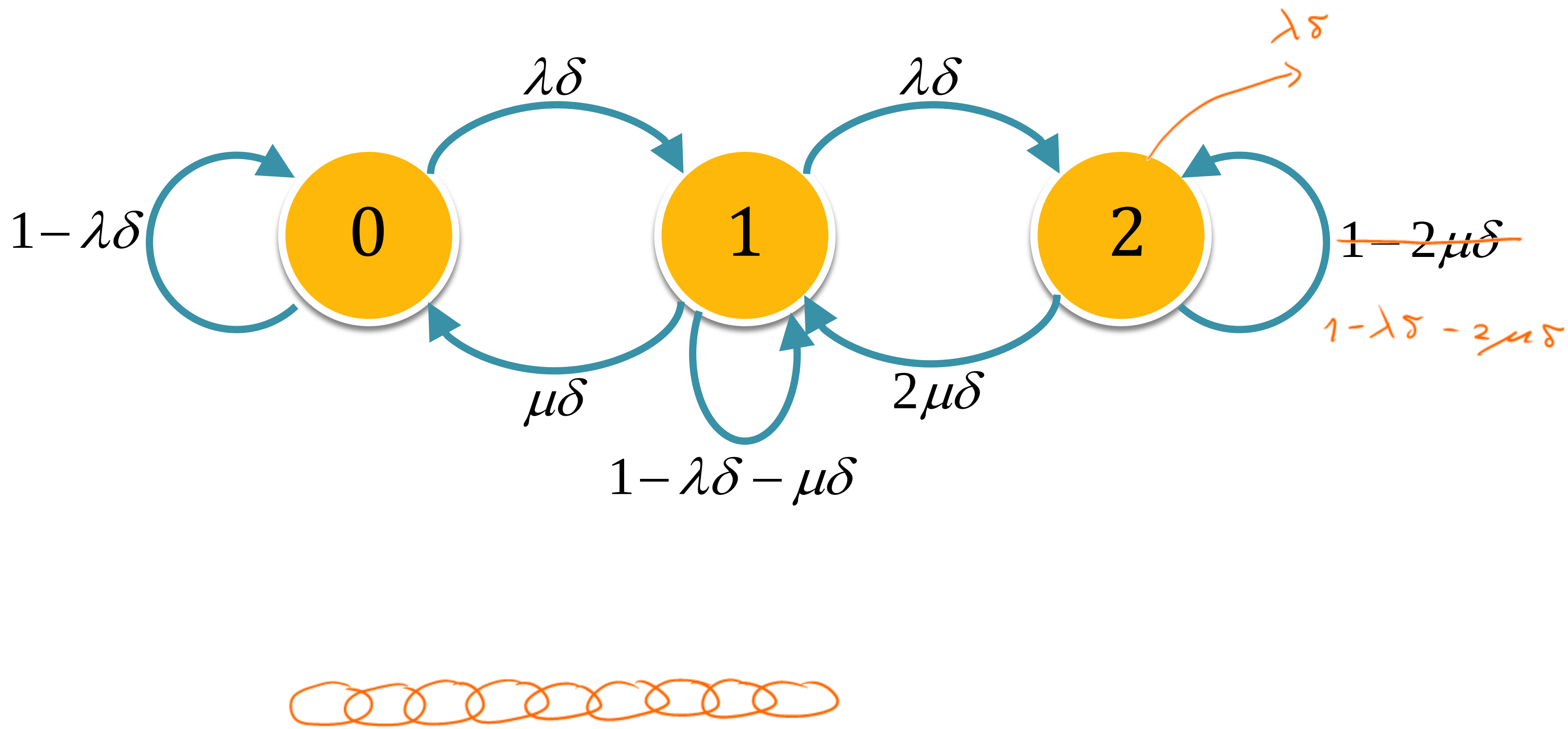
The labels on the arrows are probabilities.

P[0 new call request] ≈ 1 - $\lambda\delta$
P[1 new call request] ≈ $\lambda\delta$

P[0 old-call end] ≈ 1 - $k\mu\delta$
P[1 old-call end] ≈ $k\mu\delta$

Small slot Analysis: Markov Chain

- Case: m = 2



Markov Chain

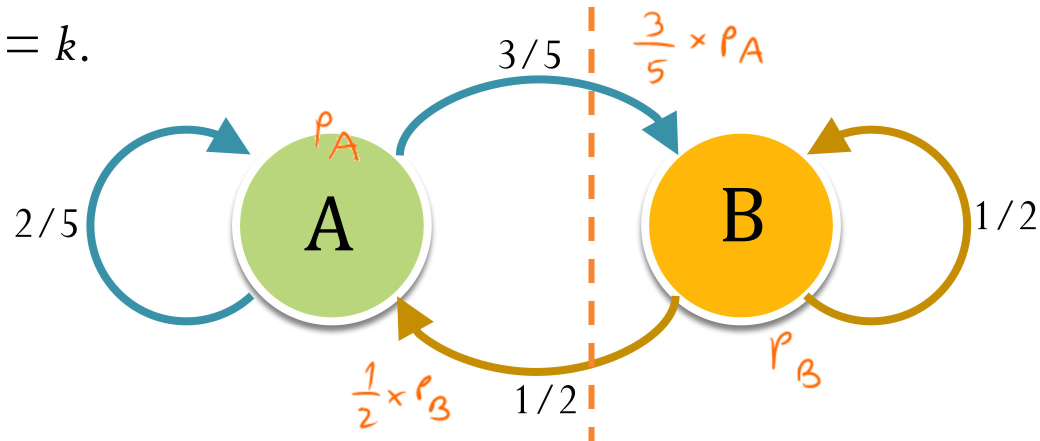
- One important property: **Memoryless**
  - It retains no memory of where it has been in the past.
  - Only the current state of the process can influence where it goes next.
- Very similar to the ***state transition diagram*** in digital circuits.
  - If the system is currently at a particular state, where would it go next on the next time slot?
  - In digital circuit, the labels on the arrows indicate the input/control signal.
  - Here, the labels on the arrows indicate transition probabilities.
- We will focus on **discrete time Markov chain**.

Global Balance Equations

- Easier approach for finding the long-term probabilities

$$P = \begin{bmatrix} 2/5 & 3/5 \\ 1/2 & 1/2 \end{bmatrix}$$

Let p_k be the long-term probability that $K = k$.



$$p_A \times \frac{3}{5} = p_B \times \frac{1}{2}$$

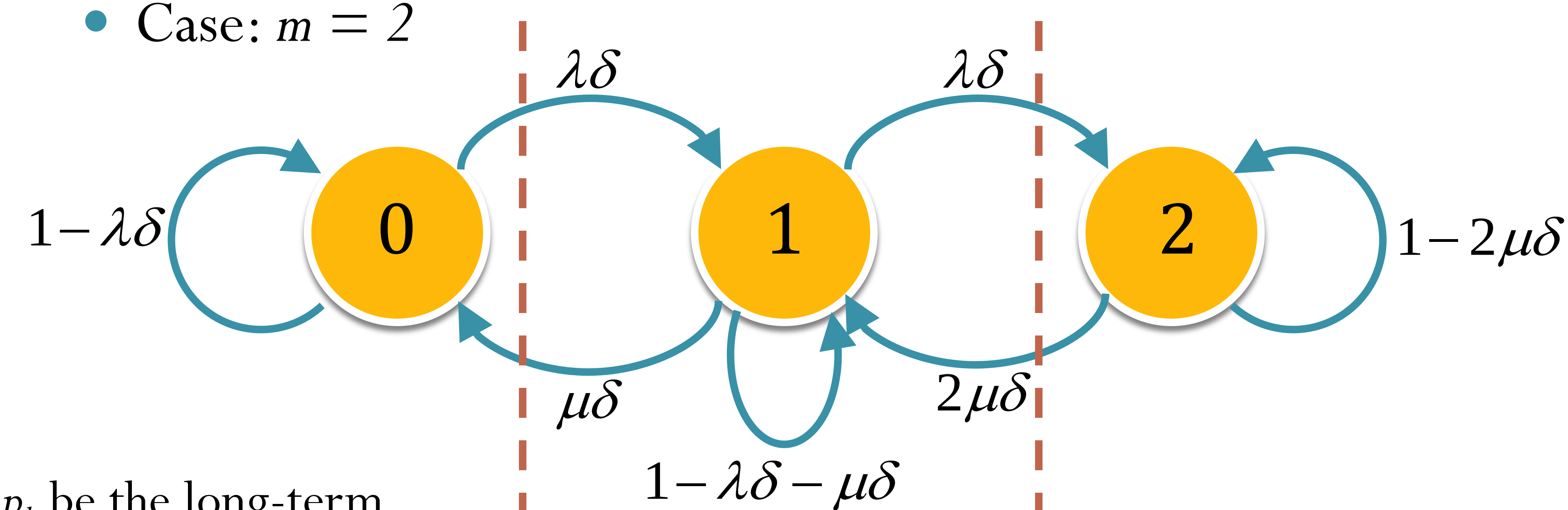
$p_A + p_B = 1$

$p_A = 0.45$

$p_B = 0.55$

Small Slot Analysis: Markov Chain

- Case: $m = 2$



Let p_k be the long-term probability that $K = k$.

Global Balance equations

$$\lambda\delta p_0 = \mu\delta p_1 \qquad \lambda\delta p_1 = 2\mu\delta p_2$$

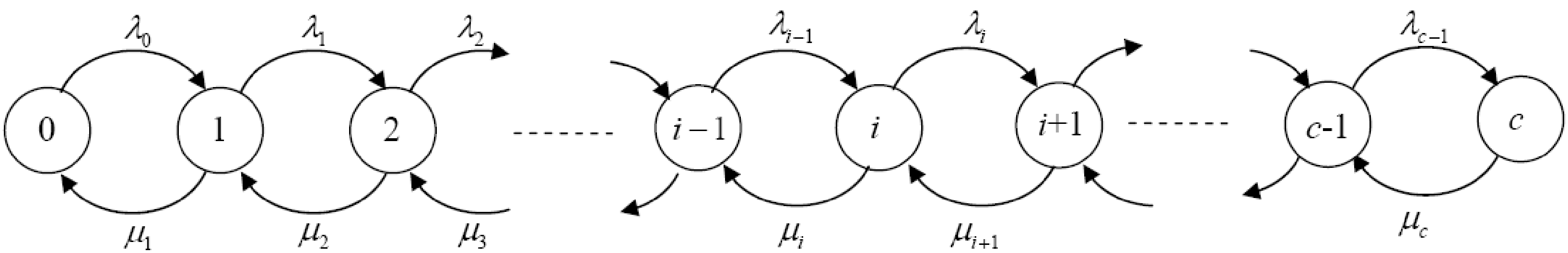
$$p_0 + p_1 + p_2 = 1$$

$$p_0 = \frac{1}{1 + A + \frac{A^2}{2}}, \quad p_1 = A p_0, \quad p_2 = \frac{1}{2} A^2 p_0$$

$$p_b = p_m$$

Truncated birth-and-death process

- Continuous-time Markov chain
- More general than M/M/m/m



The stationary PMF always exists and is given by $p_i = \dfrac{R_i}{\sum_{j=0}^{c} R_j}$ where $r_j = \dfrac{\lambda_{j-1}}{\mu_j}$,

$R_j = r_j r_{j-1} \cdots r_1$ for $j = 1, 2, \ldots, c$, and $R_0 = 1$.