Friday, December 17, 2010
11:30 AM

Lecture 11 (Dec 17)

Erlang B Formula

Review: M/M/m/m Assumption

- Poisson Process with rate λ $\rightarrow$ calls generation initialization
- Call durations: average $\frac{1}{\mu} = H$

  i.i.d. exponential

$$A = \frac{\lambda}{\mu}$$

Traffic intensity

To see where Erlang B formula comes from, we again use small-slot approximation.

Let's consider this small slot.

At the beginning of this slot let's assume $K = 5$

what kind of info. do we need to determine K at the end of the slot.

① How many new calls?

② How many old calls end?

We can answer question ① right away from properties of Poisson process:

There can be at most one new call which happens with probability

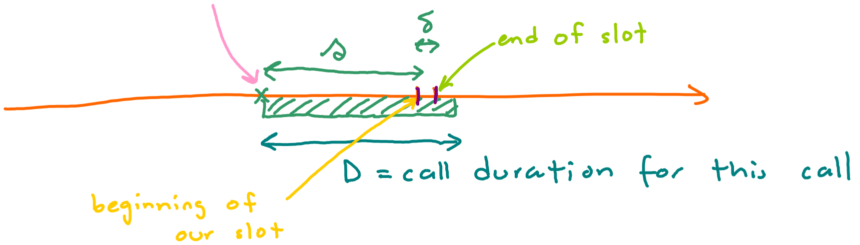
$$P_1 = \lambda\delta.$$

For question ②, we will use the small-slot approximation again.

Concentrate on one slot.

Let's consider a particular call that is still on-going in our small slot.

start time of this call



If we have to consider this user on our slot, we know that

$$D > s.$$

If $D > s+\delta$, then the call will <u>not</u> end in our slot.

So we need to find

$$P[D > s+\delta \mid D > s] = \frac{P[D > s+\delta \text{ and } D > s]}{P[D > s]}$$

$$= \frac{P[D > s+\delta]}{P[D > s]} = \frac{e^{-\mu(s+\delta)}}{e^{-\mu(s)}} = e^{-\mu\delta}$$

For exp. r.v.

$$D \sim \mathcal{E}(\mu)$$

$$f_D(d) = \mu e^{-\mu d}, \quad d > 0$$

$$P[D > x] = \int_x^\infty \mu e^{-\mu d}\, dd = e^{-\mu x}$$

Suppose, at the start time of our slot, we have k calls

① Probability that <u>none</u> of them ends is (all of them survive)

$$\left(e^{-\mu\delta}\right)^k = e^{-\mu k\delta} = 1 - \mu k\delta$$

$$e^x \approx 1 + x$$

② Probability that <u>exactly one</u> of them ends

$$k\underbrace{\left(1-e^{-\mu\delta}\right)}\left(e^{-\mu\delta}\right)^{k-1} = k\mu\delta\left(1-\mu(k-1)\delta\right)$$

$1-(1-\mu\delta)$ $\quad = k\mu\delta - \ldots$

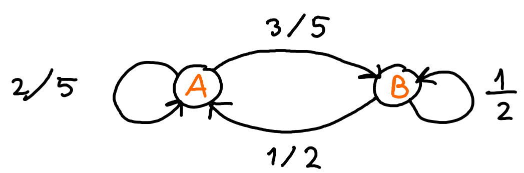
Because $(1-\mu k\delta) + (\mu k\delta) = 1$ already,

we conclude that only two cases above can happen!!

Lecture 12 (Dec 21)

See slides.

Simple Markov Chain $\quad (m=1)$



B A A B B A B B B B A A B B ....

10^5 trials

B
B
B
B
B
B
B

Slot # 1

A	B
0	100,000

Slot # 2

50,000	50,000

$\frac{2}{5} \times$

Slot # 3

20,000 + 25,000	25,000 + 30,000
45,000	55,000

Slot # 4

45,500	54,500

Slot # 5

45,450	54,550

Slot # 6

45,455

54,545

slot # 7

45454.5

54545.5