ECS455 Chapter 2

Cellular Systems

2.4 Traffice Handling Capacity and Erlang B Formula

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Capacity Concept: A Revisit

Trunking

- **Trunking**
- Allow a large number of users to share the relatively small number of channels in a cell by providing access to each user, on demand, from a pool of available channels.
- Exploit the statistical behavior of users
- Each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

Common Terms

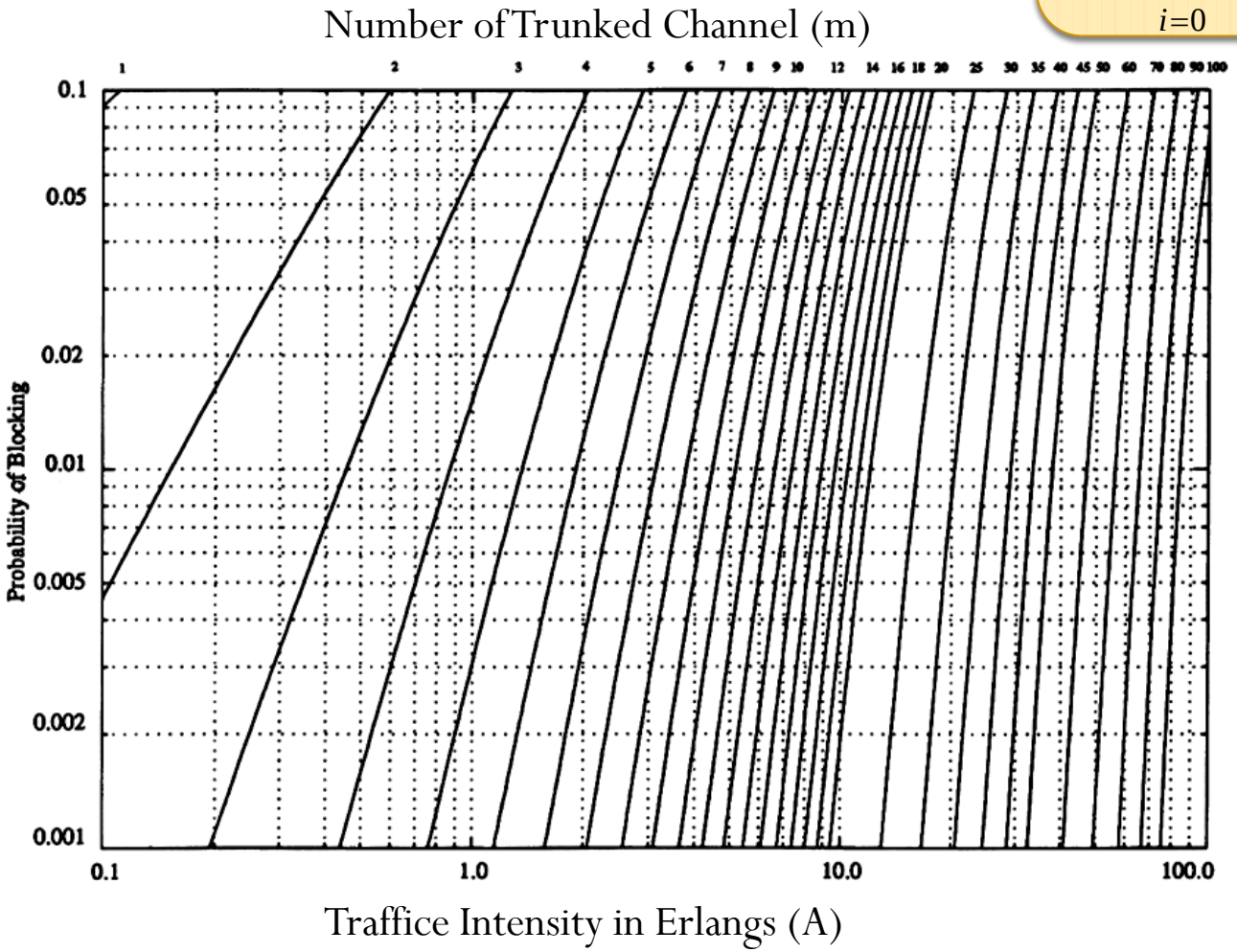
- **Traffic Intensity**: Measure of channel time utilization, which is the average channel occupancy measured in Erlangs.
  - This is a dimensionless quantity and may be used to measure the time utilization of single or multiple channels.
  - Denoted by A.
- **Holding Time**: Average duration of a typical call. Denoted by $H = 1/\mu$.
- **Blocked Call**: Call which cannot be completed at time of request, due to congestion. Also referred to as a **lost call**.
- **Grade of Service (GOS)**: A measure of congestion which is specified as the probability of a call being blocked (for Erlang B).
  - The AMPS cellular system is designed for a GOS of 2% blocking. This implies that the channel allocations for cell sites are designed so that 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.
- **Request Rate**: The average number of call requests per unit time. Denoted by λ.

M/M/m/m Assumption

- **Blocked calls cleared**
  - Offers no queuing for call requests.
  - For every user who requests service, it is assumed there is no setup time and the user is given immediate access to a channel if one is available.
  - If no channels are available, the requesting user is blocked without access and is free to try again later.
- **Calls arrive as determined by a *Poisson process*.**
- There are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time.
- There are an infinite number of users (with finite overall request rate).
  - The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.
- **The duration of the time that a user occupies a channel is exponentially distributed**, so that longer calls are less likely to occur.
- There are m channels available in the trunking pool.
  - For us, m = the number of channels for a cell (C) or for a sector

Erlang B Chart

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$



Example 1

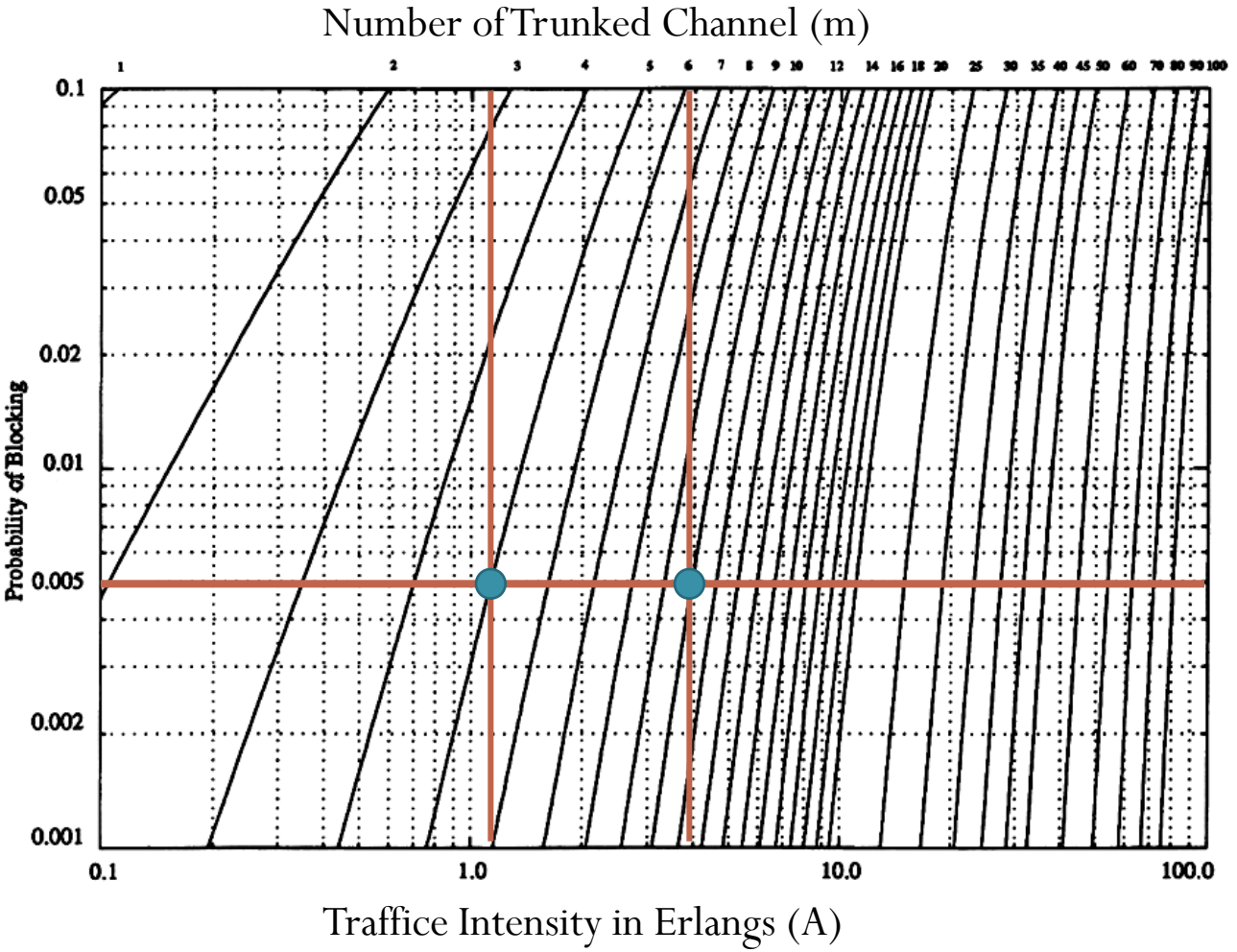
- How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

  (a) 5

  (b) 10

- Assume each user generates 0.1 Erlangs of traffic.

Erlang B

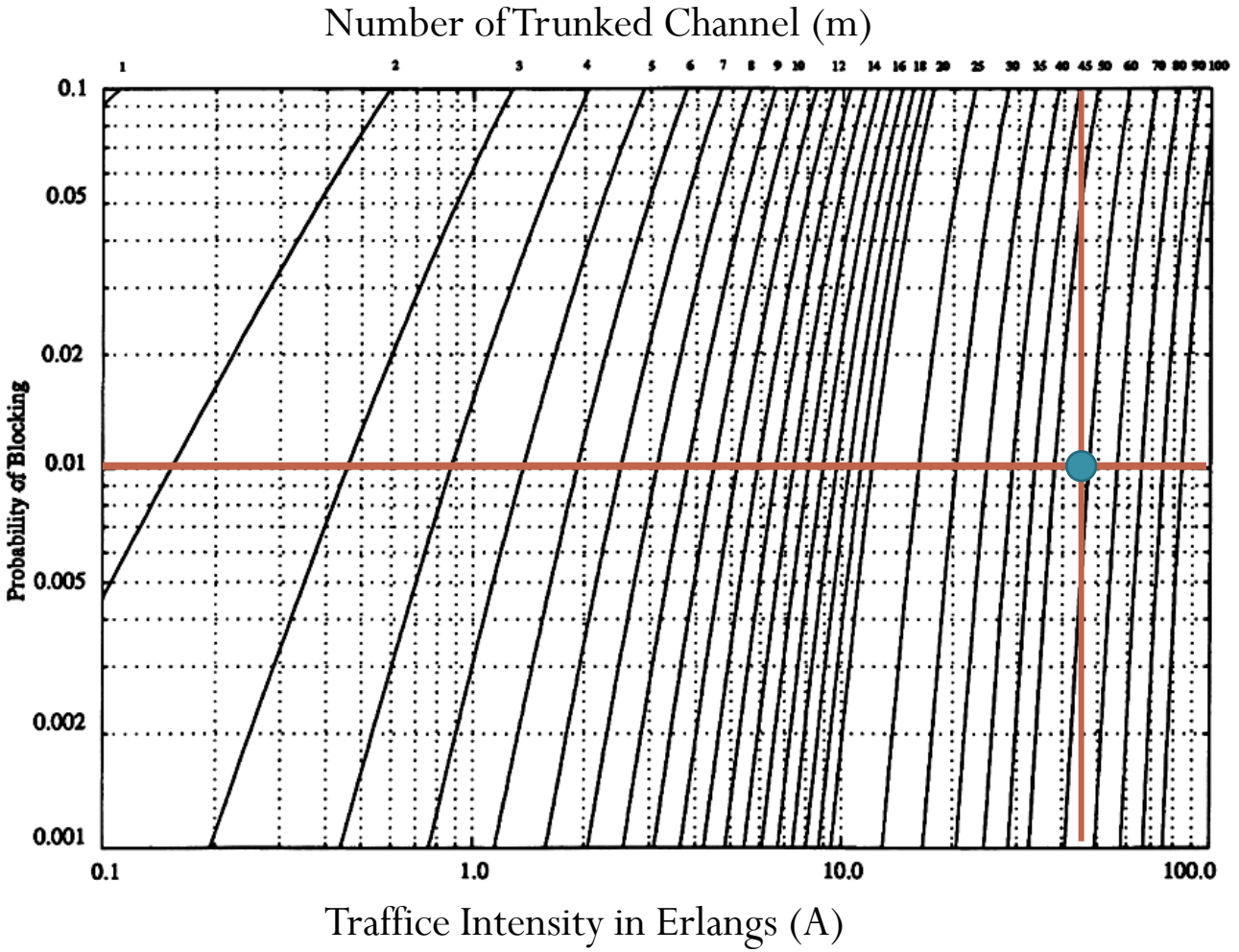

Number of Trunked Channel (m)
1
2
3
4
5
6
7
8
9
10
12
14
16
18
20
25
30
35
40
45
50
60
70
80
90
100
0.1
0.05
0.02
0.01
0.005
0.002
0.001
Probability of Blocking
0.1
1.0
10.0
100.0
Traffice Intensity in Erlangs (A)

Example 2.1

- Consider a cellular system in which
  - an average call lasts two minutes
  - the probability of blocking is to be no more than 1%.
- If there are a total of 395 traffic channels for a seven-cell reuse system, there will be about 57 traffic channels per cell.
- From the Erlang B formula, can handle 44.2 Erlangs or **1326 calls per hour**.

[Rappaport, 2002, Ex 3.9, p 92]

Erlang B

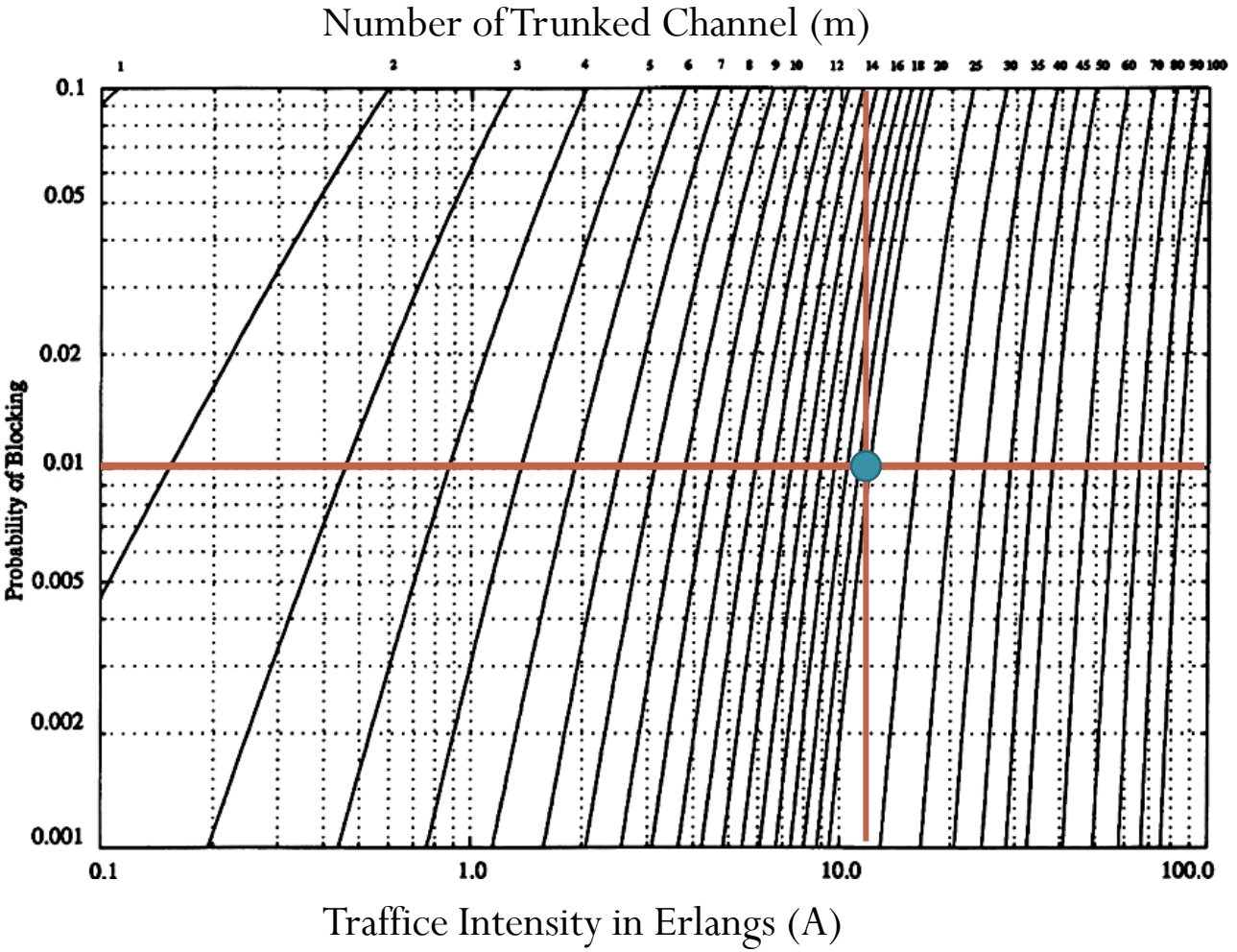

Number of Trunked Channel (m)
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 25 30 35 40 45 50 60 70 80 90 100
0.1
0.05
0.02
0.01
0.005
0.002
0.001
Probability of Blocking
0.1
1.0
10.0
100.0
Traffice Intensity in Erlangs (A)

Example 2.2

- Now employing **120° sectoring**, there are only 19 channels per antenna sector (57/3 antennas).
- For the same probability of blocking and average call length, each sector can handle 11.2 Erlangs or 336 calls per hour.
- Since each cell consists of three sectors, this provides a cell capacity of $3 \times 336 = 1008$ **calls per hour**, which amounts to a 24% decrease when compared to the unsectored case.
- Thus, sectoring decreases the **trunking efficiency** while improving the S/I for each user in the system.

[Rappaport, 2002, Ex 3.9, p 92]

Erlang B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60
70
80
90
100
0.1
0.05
0.02
0.01
0.005
0.002
0.001
Probability of Blocking
0.1
1.0
10.0
100.0
Traffice Intensity in Erlangs (A)

Erlang B Trunking Efficiency

Table 3.4 Capacity of an Erlang B System

Number of Channels *C*	Capacity (Erlangs) for GOS = 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

Summary of Chapter 2: Big Picture

S = total # available duplex radio channels for the system

Frequency reuse with **cluster size N**

"Capacity" $C = \frac{A_{total}}{A_{cell}} \times \frac{S}{N}$

Tradeoff $\frac{S}{I} \approx \frac{kR^{-\gamma}}{K \times (kD^{-\gamma})} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma} = \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$

Path loss exponent

m = # channels allocated to each cell.

Omni-directional: K = 6
120° Sectoring: K = 2
60° Sectoring: K = 1

Trunking

Call blocking probability $P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}$.

Erlang-B formula

λ = Average # call attempts/requests per unit time

A = traffic intensity or load [Erlangs] = $\frac{\lambda}{\mu}$

$\frac{1}{\mu} = H$ = Average call length

Example 3 (1)

- 20 MHz of total spectrum.
- Each simplex channel has 25 kHz RF bandwidth.
- The number of duplex channels:

$$S = \frac{20 \times 10^6}{2 \times 25 \times 10^3} = 400 \text{ channels}$$

- $\gamma = 4$
- Design requirements:
  - SIR $\geq$ 15 dB
  - $P_b \leq 5\%$

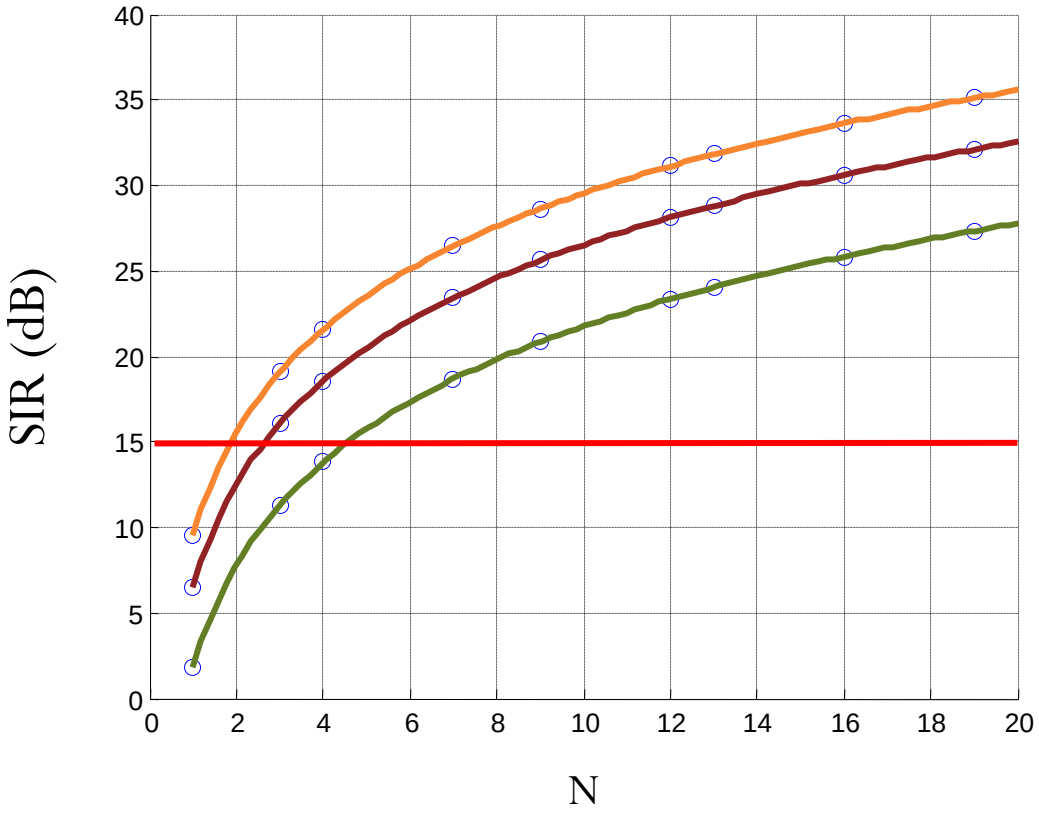
HW2 Q3

Example 3 (2)

- SIR $\geq$ 15 dB

$$\frac{S}{I} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

```
clear all; close all;
y = 4;
figure; grid on; hold on;
for K = [1,2,6]
   N = [1, 3, 4, 7, 9, 12, 13, 16, 19];
   SIR = 10*log10(1/K*((sqrt(3*N)).^y))
   plot(N,SIR,'o')
   N = linspace(1,20,100);
   SIR = 10*log10(1/K*((sqrt(3*N)).^y));
   plot(N,SIR)
end
```



K = 1 → N = 3

K = 2 → N = 3

K = 6 → N = 7

HW2 Q5

Example 3 (3)

	Omidirectional	Sectoring (120°)	Sectoring (60°)
K	6	2	1
N	7	3	3
SIR [dB]	18.7	16.1	19.1
#channels/cell	400/7 = 57	400/3 = 133	400/3 = 133
#sectors	1	3	6
#channels/sector	57	133/3 = 44	133/6 = 22
A [Erlangs]/sector	51.55	38.56	17.13
A [Erlangs]/cell	51.55	38.56×3 = 115.68	17.13×6 = 102.78
#users/cell	18558	41645	37001

Assume that each user makes 2 calls/day and 2 min/call on average → 1/360 Erlangs.

Make sure that you understand where numbers in the table come from!