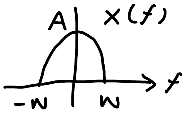
Lecture 3 (Nov 16)

C: Modulation

f_c ← Carrier frequency.

$\omega_c = 2\pi f_c$ ← carrier angular frequency.



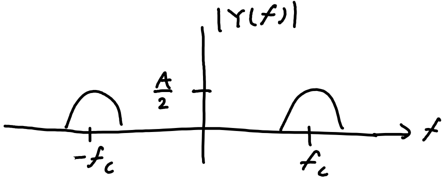
← baseband signal

$y(t) =$

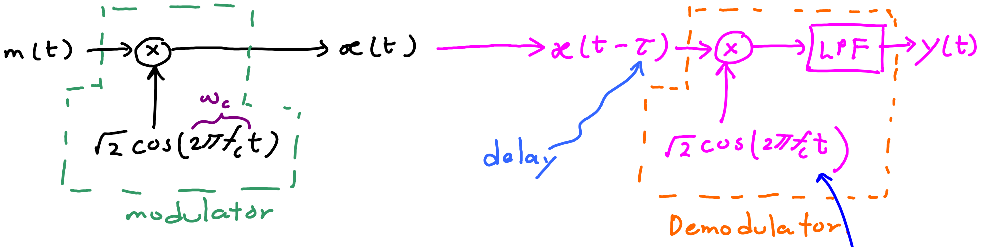
$x(t) \times \cos(\omega_c t + \phi) = x(t)\,\dfrac{e^{j(\omega_c t+\phi)} + e^{-j(\omega_c t+\phi)}}{2}$

$$= \frac{1}{2}\left(x(t)e^{j\omega_c t}e^{j\phi} + x(t)e^{-j\omega_c t}e^{-j\phi}\right)$$

$$\xrightarrow[\mathcal{F}]{} \frac{1}{2}\left(X(f-f_c)e^{j\phi} + X(f+f_c)e^{-j\phi}\right) = Y(f)$$



D: Basic DSB-SC
(Double-sideband with suppressed carrier)



In HW1, $\sqrt{2}\cos(2\pi f_c t - \theta)$

Q: $y(t) = ?$ (Hope that it is $m(t-\tau)$.)

$x(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$

$\Big\downarrow$ replace t by $t-\tau$

$x(t-\tau) = m(t-\tau)\sqrt{2}\cos(2\pi f_c (t-\tau))$

$= m(t-\tau)\sqrt{2}\cos(2\pi f_c t - \underbrace{2\pi f_c \tau}_{\phi})$

$x(t-\tau) \times \sqrt{2}\cos(2\pi f_c t)$

$= m(t-\tau)\sqrt{2}\cos(2\pi f_c t - \phi)\sqrt{2}\cos(2\pi f_c t)$

$= m(t-\tau)\left(\cos(2\pi 2 f_c t - \phi) + \cos(+\phi)\right)$

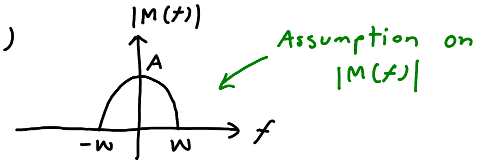
$2\cos A \cos B$
$= \cos(A+B) + \cos(A-B)$

Lecture 4 (Nov 18)

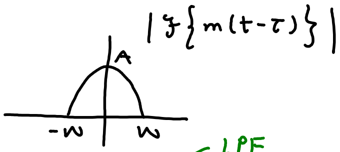
Lecture 4 (Nov 19)

$$= m(t-\tau)\cos\Big(2\pi\overbrace{(2f_c)}^{2w_c}t - \phi\Big) + m(t-\tau)\cos(\phi)$$

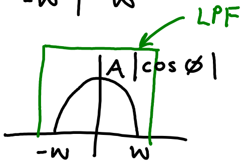
$m(t) \xrightarrow{\mathcal{F}} M(f)$



$m(t-\tau) \xrightarrow[\mathcal{F}]{}$

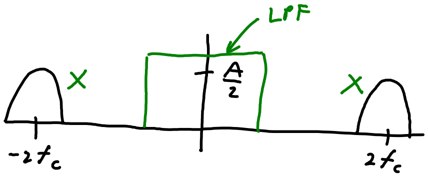


$m(t-\tau)\cos(\phi) \xrightarrow[\mathcal{F}]{}$



✓

$\underbrace{m(t-\tau)}\cos(2\pi(2f_c)t-\phi) \xrightarrow[\mathcal{F}]{}$



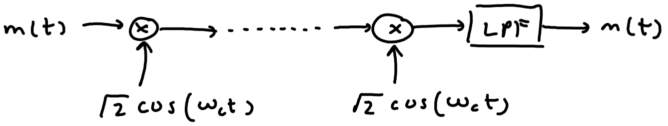
Conclusion : $y(t) = m(t-\tau)\cos(\phi)$

$= m(t-\tau)\cos(2\pi f_c\tau)$

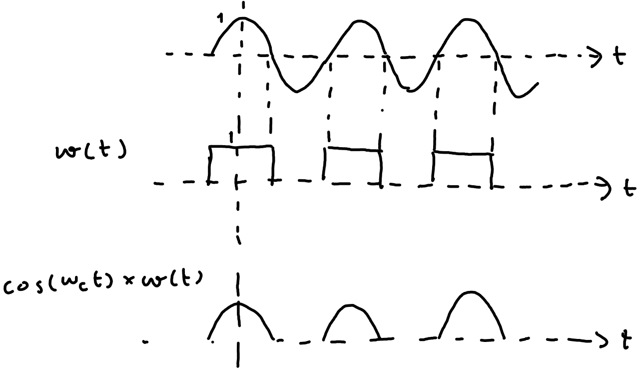
$$m(t) \rightarrow \boxed{\text{System}} \rightarrow m(t-\tau)\underbrace{\cos(2\pi f_c\tau)}_{\text{b/w } -1 \text{ and } 1 \text{ include } 0.}$$

Q: How can we fix this??

Summary :



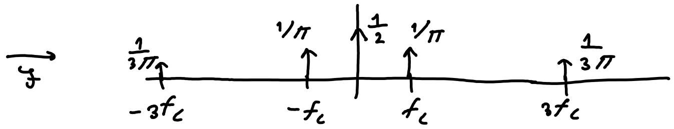
E. Switching Demodulator

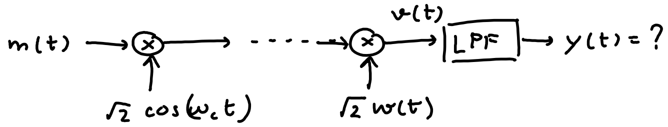


$$w(t) = 1\Big[\cos(\omega_c t) \geq 0\Big] \underset{\uparrow}{=} \frac{1}{2} + \frac{2}{\pi}\Big(\cos(\omega_c t) - \frac{1}{3}\cos(3\omega_c t) + \frac{1}{5}\cos(5\omega_c t) - \frac{1}{7}\cos(7\omega_c t) + \cdots\Big)$$

$$w(t) = 1\left[\cos(\omega_c t) \geq 0\right] \underset{\uparrow}{=} \frac{1}{2} + \frac{2}{\pi}\left(\cos(\omega_c t) - \frac{1}{3}\cos(3\omega_c t) + \frac{1}{5}\cos(5\omega_c t) - \frac{1}{7}\cos(7\omega_c t) + \cdots\right)$$

Fourier Series





$$v(t) = m(t)\sqrt{2}\cos(\omega_c t)\sqrt{2}\left(\frac{1}{2} + \frac{2}{\pi}\left(\cos(\omega_c t) - \frac{1}{3}\cos(3\omega_c t) + \frac{1}{5}\cos(5\omega_c t) \cdots\right)\right)$$

$$= m(t)\cos(\omega_c t) + \underbrace{m(t)\,2\cos(\omega_c t)\frac{2}{\pi}\cos(\omega_c t)} + \cdots$$

LPF LPF

$$= m(t)\frac{2}{\pi}\left(\cos(2\omega_c t) + \cos(0)\right)$$

1

$$= m(t)\frac{2}{\pi} + m(t)\frac{2}{\pi}\cos(2\omega_c t)$$

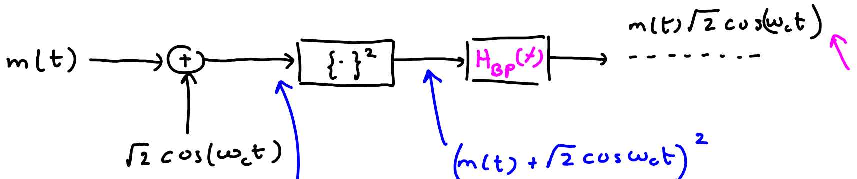
LPF

$$y(t) = \frac{2}{\pi}m(t)$$

$\cos(\omega_c t) \times w(t)$ → This can be implemented by HWF
(half-wave rectifier)
(diodes)

Caution : To use this, need $m(t) \geq 0$.

F. Square Modulator.

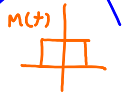


$m(t) + \sqrt{2}\cos(\omega_c t)$

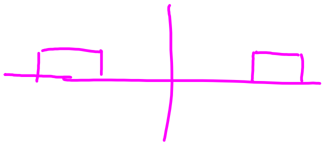
$$\left(m(t) + \sqrt{2}\cos\omega_c t\right)^2$$

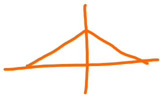
$$= m^2(t) + 2m(t)\sqrt{2}\cos\omega_c t + 2\cos^2\omega_c t$$

0 band limited to 2W f_c 0 $2f_c$



$\mathcal{F}\{m^2(t)\}$





$$H_{BP}(f) = \begin{cases} 1/2, & |f - f_c| \leq W \\ 1/2, & |f + f_c| \leq W \\ 0, & \text{otherwise} \end{cases}$$

HW1 Due Tuesday (Nov 23)