Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 4

Semester/Year: 2/2016

Course Title: Mobile Communications

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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

Due date: Not Due

Questions

1. Consider a system which has 3 channels. We would like to find the blocking probability via the Markov chain method. For each of the following models, **draw the Markov chain** via discrete time approximation. Find (1) the **steady-state probabilities** and (2) the long-term **call blocking probability**.
   a. ***Erlang B*** model: Assume that the *total* call request rate is 10 calls per hour and the average call duration is 12 mins.

b. ***Engset*** model: Assume that there are 5 users. The call request rate for *each* user is 2 calls per hour and the average call duration is 12 mins.
c. ***Engset*** model: Assume that there are 100 users. The call request rate for *each* user is 0.1 calls per hour and the average call duration is 12 mins.

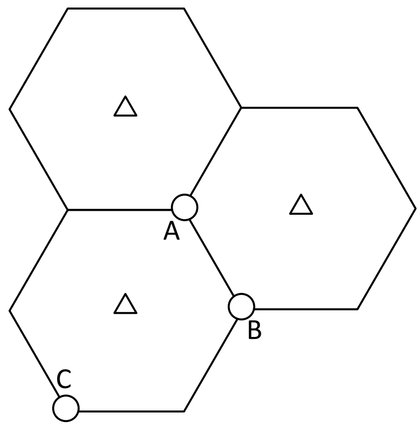
Extra Questions

Here are some extra questions for those who want more practice.

2. **[M2009/1Q2]** In this question, we consider the SIR value when the cluster size $N = 1$. Sectoring is not used. Suppose this cellular system has only three base stations. They are marked by triangles located at the centers of three cells in the figure below. Assume that they transmit the same power level.

   User A (mobile station A), user B (mobile station B), and user C (mobile station C) are currently associated with the lower-left base station. The locations of these users are marked by the circles in the figure.

   Assume a path loss exponent of $\gamma = 4$. Do not approximate distance values.



   a. Find the signal-to-interference ratio (in dB) for user A.

   b. Find the signal-to-interference ratio (in dB) for user B.

   c. Find the signal-to-interference ratio (in dB) for user C.

3. **[M2009/1Q5]** A function $\text{ErlangB}(m, A)$ is defined by

$$\text{ErlangB}(m, A) = \frac{\frac{A^m}{m!}}{\sum_{k=0}^{m} \frac{A^k}{k!}}$$

a. (2 pt) Compare $\text{ErlangB}(1000, 2000)$ and $\text{ErlangB}(1000, 2001)$.
Which one is larger?

b. (2 pt) Compare $\text{ErlangB}(1000, 2000)$ and $\text{ErlangB}(1001, 2000)$.
Which one is larger?

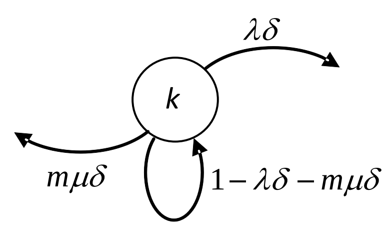
c. (1 pt) Suppose

$$\text{ErlangB}(1000, A_1) = \text{ErlangB}(2000, A_2) = \text{ErlangB}(3000, A_3) = 0.05$$

Let $A_4 = A_1 + A_2$.
Compare A_3 and A_4. Which one is larger?

4. **[M2009/1Q4]** There are 1000 users subscribed to a cellular system. The call request rate for each user is 2 call requests per week. For each call, the average call duration is 1 min. If the system has only two channels and it is used to support the whole 1000 users, what is the blocking probability?

5. Consider another modification of the M/M/m/m (Erlang B) system. (There are infinite users) Assume that there is a queue that can be used to hold all requested call which cannot be immediately assigned a channel. This is referred to as an M/M/m/∞ or simply M/M/m system. We will define state k as the state where there are k calls in the system. If $k \le m$, then all of these calls are ongoing. If $k > m$, then m of them are ongoing and k-m of them are waiting in the queue.
   Assume that the total call request process is Poisson with rate λ and that the call durations are i.i.d. exponential random variables with expected value $1/\mu$.
   Also assume that $\frac{\lambda}{\mu} < m$.
   a. **<u>Draw the Markov chain</u>** via discrete time approximation. Don't forget to indicate the transition probabilities (in terms of λ, μ, and δ) on the arrows.
      Hint: there are infinite number of states. The transition probabilities for state k which is < m are the same as in the M/M/m/m system. For $k \ge m$, the transition probabilities are given below:



      Explain why the above transition probabilities make sense.
   b. Find the **<u>steady-state probabilities</u>** using balance equations
   c. Find the long-term **<u>delayed call probability</u>** (the probability that a call request occurs when all m channels are busy and thus has to wait).
      Hint: This will be a summation of many steady-state probabilities. When you simplify your answer, the final answer should be

$$\frac{A^m}{A^m + m!\left(1-\frac{A}{m}\right)\sum_{k=0}^{m-1}\frac{A^k}{k!}}.$$

6. Consider the Engset model
   a. Show that the steady-state probabilities for the Engset model are given by

$$p_i = \frac{\binom{n}{i} A_u^i}{\sum_{k=0}^{m} \binom{n}{k} A_u^k} = \frac{\binom{n}{i} A_u^i}{z(m,n)},\ 0 \le i \le m,$$

   where $z(m,n) = \sum_{k=0}^{m} \binom{n}{k} A^k$.

   b. Express p_m (time congestion) in the form $p_m = 1 - \frac{z(m-c,n)}{z(m,n)}$.

   What is the value of c?

   Hint: c is an integer.

   c. Show that the blocked call probability is given by $P_b = \frac{(n-m)\binom{n}{m} A_u^m}{\sum_{k=0}^{m} (n-k)\binom{n}{k} A_u^k}$.

   d. The blocked call probability can be rewritten in the form $P_b = 1 - \frac{z(m-c_1,n-c_2)}{z(m-c_3,n-c_4)}$.

   Find c_1, c_2, c_3, c_4.

   Hint: c_1, c_2, c_3, c_4 are all integers.

   e. Suppose $m = n-1$. Simplify the expression for P_b.

   Hint: Your answer should be of the form $\left(g\left(A_u\right)\right)^m$ for some function *g* of *A*$_u$.