Sirindhorn International Institute of Technology
Thammasat University at Rangsit
School of Information, Computer and Communication Technology

ECS 455: Problem Set 3

Semester/Year: 2/2016

Course Title: Mobile Communications
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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

Due date: March 17, 2017 (Friday), 4:30 PM

Instructions

1. (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of every submitted sheet.
2. (1 pt) For each part, write your explanation/derivation and answer in the space provided.
3. (8 pt) It is important that you try to solve all non-optional problems.
4. Late submission will be heavily penalized.

Questions

1. Consider a trunked system. Assume that the state of the system is $K(t) = 0$ for $t < 0$. The provided table shows the time of the (new) call requests and the corresponding call durations (if the calls can be made).
   a. Suppose $m = 1$.
      i. Plot the system state $K(t)$ from t = 0 to 120.

Call#	Call Request Time	Call Duration (if the call can be made)
1	7	86
2	28	32
3	35	218
4	37	78
5	44	70
6	53	11
7	77	47
8	85	318
9	116	115
10	117	66

ii. Indicate the call# of the calls that are blocked.

b. Suppose $m = 2$.
   i. Plot the system state $K(t)$ from $t = 0$ to 120.

   ii. Indicate the call# of the calls that are blocked.

2. Consider a single base station mobile radio system. Suppose the total call request rate is three calls per hour, each call lasting an average of 5 minutes. Assume M/M/m/m model.
   a. What is the probability that there are exactly two call requests during the time interval from 9 AM to 11 AM?

   b. Consider two time intervals:

      I_1 = the time interval from 9 AM to 11 AM, and

      I_2 = the time interval from 1 PM to 3 PM.

      i. Find the probability that there are exactly two call requests in I_1 and there are exactly three call requests in I_2.

ii. Suppose we know that there are exactly two call requests in I_1. Find the probability that there are exactly three call requests in I_2.

c. Suppose we know that a call request happens at 10 AM.

i. Suppose there is an available channel for this call.

I. Find the probability that it is still **ongoing** at 11 AM.

II. Find the probability that it **ends** before 10:05 AM.

III. Suppose the call is still ongoing at 10:05 AM. What is the probability that it is still ongoing at 10:06AM?

ii. Find the probability that the next call request happens before 11AM.

3. Complete the following M/M/m/m description with the following terms:

(I) Bernoulli	(II) binomial	(III) exponential
(IV) Gaussian	(V) geometric	(VI) Poisson

The Erlang B formula is derived under some assumptions. Two important assumptions are (1) the call request process is modeled by a/an ______(A)________ process and (2) the call durations are assumed to be i.i.d. ______(B)________ random variables. For the call request process, the times between adjacent call requests can be shown to be i.i.d. ______(C)________ random variables. On the other hand, if we consider non-overlapping time intervals, the numbers of call requests in these intervals are ______(D)________ random variables.

In order to analyze or simulate the system described above, we consider slotted time where the duration of each time slot is small. This technique shifts our focus from continuous-time Markov chain to discrete-time Markov chain. In the limit, for the call request process, only one of the two events can happen during any particular slot: either (1) there is one new call request or (2) there is no new call request. When the slots are small and have equal length, the numbers of new call requests in the slots can be approximated by i.i.d. _____(E)_________ random variables. In which case, if we count the total number of call requests during n slots, we will get a/an _____(F)_________ random variable because it is a sum of i.i.d. _____(E)_________ random variables.

When we consider a particular time interval I (not necessarily small), the number of slots in this interval will increase as the slots get smaller. In the limit, the number of call requests in the time interval I which we approximated by a _____(F)_________ random variable before will approach a/an ______(D)________ random variable.

Similarly, if we consider the numbers of slots between adjacent call requests, these number will be i.i.d. ______(G)________ random variables. These random variables can be thought of as discrete counterparts of the i.i.d. ______(C)________ random variables in the continuous-time model.

4. (**Markov Chain**) “The Land of Oz is blessed by many things, but not by good weather. They never have two nice days in a row. If they have a nice day, they are just as likely to have snow as to have rain on the next day. If they have snow or rain, they have an even chance of having the same the next day. If there is change from snow or rain, only half of the time is this a change to a nice day.” [Grinstead and Snell, Ex 11.1][Kemeny, Snell, and Thompson, 1974]
    a. Draw the Markov chain corresponding to how the weather in the Land of Oz changes from one day to the next.
       Hint: This Markov chain will have three states: rain (R), nice (N), and snow (S).

    b. Find the corresponding (probability) transition matrix **P**.

    c. Find the steady-state probabilities by using balance equations

5. (**MATLAB**) Suppose we set

   `X = exprnd(10,1,1e6);` and `Y = X(X>5);`

   in `MATLAB`. Without using MATLAB, estimate the following quantities.

   a. ~~`average`~~`(X)`
      mean

   b. `sum([Y>6])/length(Y)`

Extra Question

Here is an optional question for those who want more practice.

6. Continue from Q4
   a. Find the steady-state probabilities by using the eigen-values & eigen-vectors (in `MATLAB`)
   b. Modify the script MarkovChain_Demo1.m discussed in class to check your answers in part (b) using simulation in `MATLAB`.
   c. Suppose it is snowing in the Land of Oz today.
      i. (*) **<u>Calculate</u>** the chance that it will be a nice day tomorrow.
      ii. (*) **<u>Calculate</u>** the chance that it will be a nice day the day after tomorrow.
      iii. **<u>Estimate</u>** the chance that it will be a nice day next year (365 days later).