Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 2

Semester/Year: 2/2016

Course Title: Mobile Communications

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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

Due date: March 1, 2017 (Wednesday), 4:30 PM

Instructions

1. (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of every submitted sheet.
2. (1 pt) For each part, write your explanation/derivation and answer in the space provided.
3. (8 pt) It is important that you try to solve all non-optional problems.
4. Late submission will be heavily penalized.

Questions

1. Consider the wireless in-door communication shown in Figure **1**. The room size is 10×10 [m^2]. Let the center of the room be at (0,0). The transmitter, denoted by a square, is located at (2,-3). The receiver, denoted by a circle, is located at (-3,3). Consider the path that travels from the transmitter, reflects from wall 1, and then reaches the receiver.
    a. Locate the coordinates of the transmitter image that the receiver sees.

    b. Find its propagation distance.

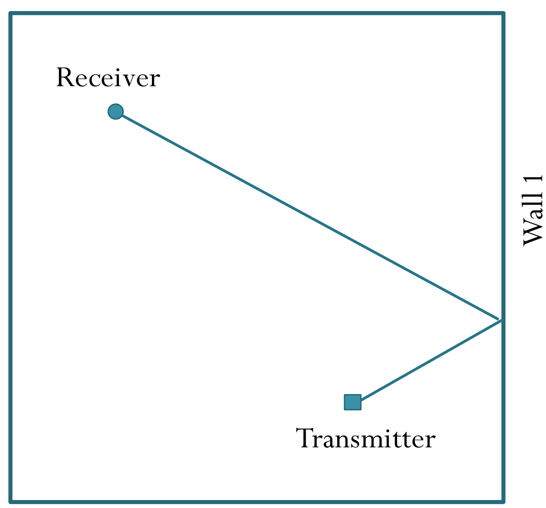

Figure 1

2. Reflection from a ground plane: Consider the propagation model in Figure **2** where there is a reflected path from the ground plane. Let r_1 be the length of the direct path. Let r_2 be the length of the reflected path (summing the path length from the transmitter to the ground plane and the path length from the ground plane to the receiver). [Tse and Viswanath, 2005, Section 2.1.5 and Exercise 2.5]

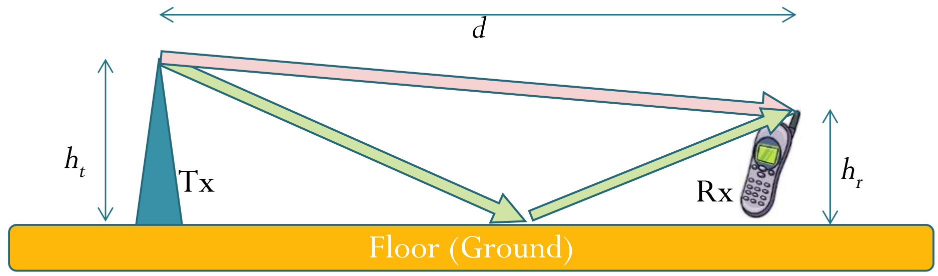

Figure 2: Illustration of a direct path and a reflected path off a ground plane [Tse and Viswanath, 2005, Figure 2.6].

a. Express r_1 and r_2 as functions of d, h_t, and h_r.

b. The plot in Figure **3** shows $r_2 - r_1$ when $h_t = 50$ m and $h_r = 2$ m. Note that the plot is done in log-log scale.

   Notice that when d is large, the graphs becomes straight lines. Estimate the slope of the line.

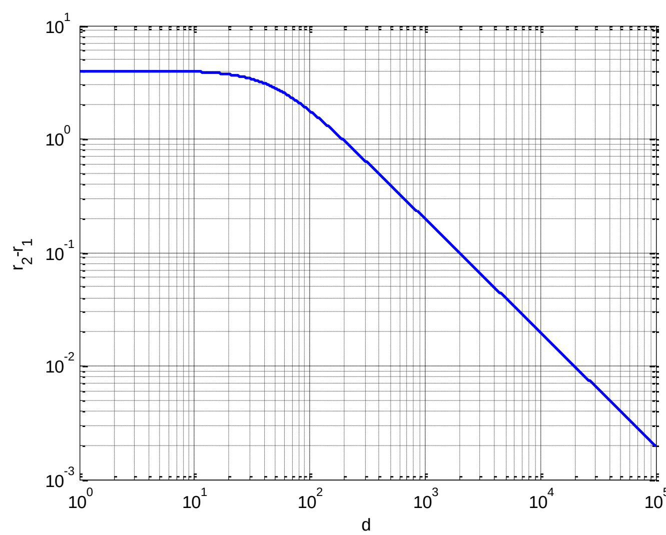

Figure 3: Log-log plot of r_2-r_1.

c. Use the slope to show that $r_2 - r_1 \propto \frac{1}{d}$ when d is large.

3. 20 MHz of total spectrum is allocated for a **duplex** wireless cellular system. Suppose each **simplex** channel uses 25 kHz RF bandwidth.
   a. Find the number of **duplex** channels.

   b. Find the total number of duplex channels per cell, if cluster size N = 4 is used.

4. In this question, we will find the unsectored (using omnidirectional antennas) SIR value when $N = 3$.
   a. Let D_i be the distance from the ith interfering co-channel cell base station. Find all the distance D_i in Figure 4. Express them as a function of R (the cell radius). Hint: $D_1 = \sqrt{13}R$.

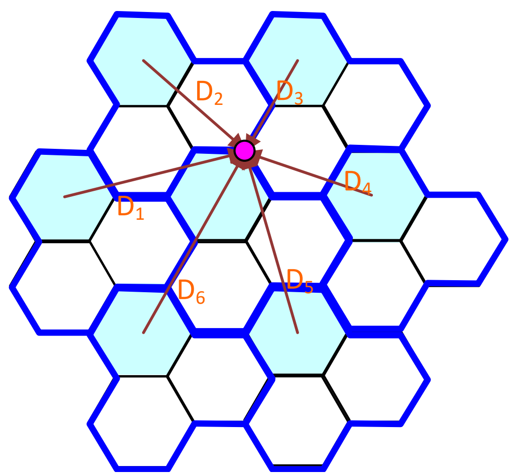

Figure 4

b. Recall that the SIR can be calculated from $\mathrm{SIR} = \dfrac{kR^{-\gamma}}{\sum_{i=1}^{K} kD_i^{-\gamma}}$ Calculate the SIR (in dB) using the D_i values that you got from part (a). Assume a path loss exponent of $\gamma = 4$.

c. Approximate all value of D_i by the **center-to-center** distance *D* between the nearest co-channel. Express *D* as a function of *R*. Recalculate the SIR (in dB) with all D_i replaced by D.

d. Compare your answers from part (b) and part (c).

5. Recall that we can approximate the SIR value by $\mathrm{SIR} = \dfrac{kR^{-\gamma}}{K \times \left(kD^{-\gamma}\right)} = \dfrac{1}{K}\left(\dfrac{D}{R}\right)^{\gamma} = \dfrac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$

   where *K* is the number of (first-tier) interfering (co-channel) base stations.
   A cellular service provider decides to use a digital TDMA scheme which can tolerate a signal-to-interference ratio of 15 dB in the worst case.
   Assume a path loss exponent of $\gamma = 4$.
   Find the optimal value of *N* for

   a. omnidirectional antennas,

   b. 120° sectoring, and

c. 60° sectoring.

6. Assume each user generates 0.1 Erlangs of traffic.
   How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?
   a. 5
   b. 15
   c. 25

7. Assume each user of a single base station mobile radio system averages three calls per hour, each call lasting an average of 5 minutes.
   a. What is the traffic intensity for each user?
   b. Find the number of users that could use the system with 1% blocking if only one channel is available.

c. Find the number of users that could use the system with 1% blocking if five trunked channels are available.

d. If the number of users you found in (c) is suddenly doubled, what is the new blocking probability of the five channel trunked mobile radio system?

Extra Questions

Here are some optional questions for those who want more practice.

8. Some possible values of cluster size is N = 1, 3, 4, or 7. Use MATLAB to find
    a. the next fifteen lowest values of N.
    b. the 1000th value of N.

9. Consider the wireless in-door communication shown in Figure 5.
   The room size is 10×10 [m^2].
   Let the center of the room be at (0,0).
   The transmitter, denoted by a square, is located at (2,-3).
   The receiver, denoted by a circle, is located at (-3,3).
   Consider the path that travels from the transmitter, reflects from wall 1, reflects from wall 2, and then reaches the receiver.
    a. Locate the coordinates of the transmitter image that the receiver sees.
    b. Find its propagation distance.

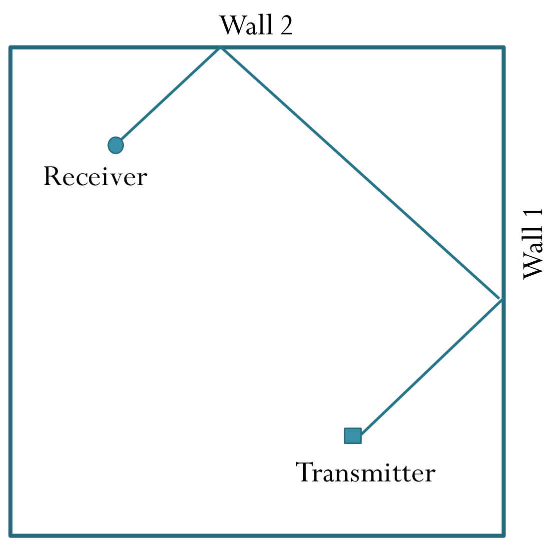

Figure 5

10. Reflection from a ground plane: Continue from Question 2.

a. Use MATLAB to plot *d* vs. $(r_2 - r_1)$ (in a similar fashion to Figure 3) for the case when $h_t = 100 \text{ m}$ and $h_r = 1 \text{ m}$.

b. Now, instead of looking at the plot(s), analytically derive the fact that when the horizontal distance *d* between the antennas becomes very large relative to their vertical displacements (h_t and h_r) from the ground, $r_2 - r_1 \propto \frac{1}{d}$.

   Hint: For small *x*, $\sqrt{1+x} \approx 1 + \frac{x}{2}$.

   Remark: This will also allow you to find the proportionality constant.

c. In class, we have shown that when the transmitted signal is $x(t) = \sqrt{2P_t}\cos(2\pi f_c t)$, the received signal is given by

$$y(t) = \frac{\alpha}{r_1}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_1}{c}\right)\right) - \frac{\alpha}{r_2}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_2}{c}\right)\right).$$

   i. Use phasor form to show that for $g(t) = a_1\cos(2\pi f_c t + \phi_1) + a_2\cos(2\pi f_c t + \phi_2)$, we have $P_g = \frac{1}{2}\left|a_1 e^{j\phi_1} + a_2 e^{j\phi_2}\right|^2$.

   ii. Show that $\frac{P_y}{P_x} = \left|\frac{\alpha}{r_1}e^{-j2\pi f_c \frac{r_1}{c}} - \frac{\alpha}{r_2}e^{-j2\pi f_c \frac{r_2}{c}}\right|^2$.

   iii. Justify each equality/approximation below:

$$\frac{P_y}{P_x} \overset{(1)}{=} \left|\frac{\alpha}{r_1} - \frac{\alpha}{r_2}e^{-j2\pi f_c \frac{r_2 - r_1}{c}}\right|^2 \overset{(2)}{\approx} \left|\frac{\alpha}{r_1} - \frac{\alpha}{r_2}e^{-j2\pi \frac{2h_t h_r}{\lambda d}}\right|^2 \overset{(3)}{\approx} \left|\frac{\alpha}{d}\left(1 - e^{-j\frac{4\pi h_t h_r}{\lambda d}}\right)\right|^2$$

$$\overset{(4)}{\approx} \left(\frac{\alpha}{d}\right)^2 \left|1 - \left(1 - j\frac{4\pi h_t h_r}{\lambda d}\right)\right|^2 \overset{(5)}{=} \frac{\alpha^2}{d^2}\left|j2\pi\frac{2h_t h_r}{\lambda d}\right|^2 = \left(\frac{4\pi\alpha h_t h_r}{\lambda d^2}\right)^2$$