2.4 Traffic Handling Capacity
and Erlang B Formula

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Thursday 13:30-14:30
Capacity Concept: A Revisit

- Q: If I have $m$ channels per cell, is it true that my cell can support only $m$ users?
- A: Yes and No
- Let’s try one example.
- How often do you make a call?
  - 3 calls a day, on average.
- How long is the call?
  - 10 mins (per call), on average.
- So, one person uses
Capacity Concept: A Revisit

- If we can “give” the time that “User 1” is idle to other users,
  - then one channel can support users!!
  
- True?
New Concepts for a New Look at Capacity

- We can let more than one user share a channel by using it at different times.
- **Blocked call** happens if a user requests to make a call when all the channels are occupied by other users.
- **Probability of (call) blocking: \( P_b \)**
  - The likelihood that a call is blocked because there is no available channel.
  - 1%, 2%, 5%

- In which case, the number of users that a cell can support can exceed \( \frac{S}{N} \).
  - How much larger depends strongly on the value of \( P_b \) that can be tolerated.
Trunking

- Allow a large number \( (n) \) of users to **share** the relatively small number \( (m) \) of channels in a cell (or a sector) by providing access to each user, **on demand**, from a **pool** of available channels.

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<th>60° Sectoring</th>
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<tbody>
<tr>
<td>#sectors/cell</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>#channels/sector</td>
<td>( m = \left\lfloor S/N \right\rfloor )</td>
<td>( m = \left\lfloor S/N /3 \right\rfloor )</td>
<td>( m = \left\lfloor S/N /6 \right\rfloor )</td>
</tr>
</tbody>
</table>

- Exploit the **statistical behavior** of users.
- Each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.
Common Terms (1)

- **Traffic Intensity**: Measure of channel time utilization (traffic load / amount of traffic), which is the average channel occupancy measured in **Erlangs**.
  - Dimensionless
  - Denoted by $A$.
- **Holding Time**: Average duration of a typical call.
  - Denoted by $H = 1/\mu$.
- **Request Rate**: The average number of call requests per unit time. Denoted by $\lambda$.
- Use $A_u$ and $\lambda_u$ to denote the corresponding quantities for one user.
- Note that $A = nA_u$ and $\lambda = n\lambda_u$ where $n$ is the number of users supported by the pool (trunked channels) under consideration.
Common Terms (2)

- **Blocked Call**: Call which cannot be completed at time of request, due to congestion.
  - Also referred to as a **lost call**.

- **Grade of Service (GOS)**: A measure of congestion which is specified as the probability of a call being blocked (for Erlang B).
  - The AMPS cellular system is designed for a GOS of 2% blocking.
    - This implies that the channel allocations for cell sites are designed so that, on average, 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.
Erlang B Formula

\[ P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}. \]

- \( m \): Number of trunked channels
- \( A \): Traffic intensity or load [Erlangs]
- \( \lambda \): Average # call attempts/requests per unit time
- \( \mu \): Average call length
- \( H \): Average call length

In MATLAB, use `erlangb(m, A)`

We use the MATLAB code from [http://infohost.nmt.edu/~borchers/erlang.html](http://infohost.nmt.edu/~borchers/erlang.html) to evaluate the Erlang B formula.
M/M/m/m Assumption

- **Blocked calls cleared**
  - No queuing for call requests.
  - For every user who requests service, there is no setup time and the user is given immediate access to a channel if one is available.
  - If no channels are available, the requesting user is blocked without access and is free to try again later.

- **Calls arrive as determined by a Poisson process.**
  - There are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time.
  - There are an infinite number of users (with finite overall request rate).
    - The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.

- **The duration of the time that a user occupies a channel is exponentially distributed**, so that longer calls are less likely to occur.

- There are $m$ channels available in the trunking pool.
  - For us, $m =$ the number of channels for a cell (C) or for a sector.
Erlang B Formula and Chart

\[ P_b = \frac{A^m}{m!} \sum_{i=0}^{m} \frac{A^i}{i!} \]

Number of Trunked Channels (m)

Traffic Intensity in Erlangs (A)

(log-log plot)
Example 1

- How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?
  
  (a) 5
  (b) 10

- Assume each user generates $A_u = 0.1$ Erlangs of traffic.
Example 1a

\[ A \approx 1 \Rightarrow n \approx 10 \text{ users} \]
Example 1b

\[ A \approx 4 \Rightarrow n \approx 40 \text{ users} \]
Example 2.1

- Consider a cellular system in which
  - an average call lasts two minutes
  - the probability of blocking is to be no more than 1%.
- If there are a total of 395 traffic channels for a seven-cell reuse system, there will be about 56 traffic channels per cell.
- From the Erlang B formula, can handle 43.31 Erlangs or 1299 calls per hour.

[Rappaport, 2002, Ex 3.9, p 92]
Example 2.1: Erlang B

Traffic Intensity in Erlangs (A)

Number of Trunked Channels (m)

$\text{erlangb}(57, 44) = 0.0094$
$\text{erlangb}(57, 44.2) = 0.0099$
$\text{erlangb}(57, 44.3) = 0.0102$
$\text{erlangb}(57, 44.5) = 0.0109$
$\text{erlangb}(57, 45) = 0.0125$
Example 2.2

- Now employing \textit{120° sectoring}, there are only \( m = 57/3 = 19 \) channels per sector.
- For the same probability of blocking and average call length, each sector can handle 11.2 Erlangs or 336 calls per hour.
- Since each cell consists of three sectors, this provides a cell capacity of \( 3 \times 336 = 1008 \text{ calls per hour} \), which amounts to a 22\% decrease when compared to the unsectored case.
- Thus, sectoring decreases the \textit{trunking efficiency} while improving the SIR for each user in the system.

[Rappaport, 2002, Ex 3.9, p 92]
Example 2.2: Erlang B

<table>
<thead>
<tr>
<th>Traffic Intensity in Erlangs (A)</th>
<th>Number of Trunked Channel (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>erlangb(19,11) = 0.0085</td>
<td>11</td>
</tr>
<tr>
<td>erlangb(19,11.2) = 0.0098</td>
<td>12</td>
</tr>
<tr>
<td>erlangb(19,11.3) = 0.0105</td>
<td>13</td>
</tr>
<tr>
<td>erlangb(19,11.5) = 0.0120</td>
<td>15</td>
</tr>
<tr>
<td>erlangb(19,12) = 0.0165</td>
<td>19</td>
</tr>
</tbody>
</table>
## Erlang B Trunking Efficiency

<table>
<thead>
<tr>
<th>Number of Channels ( m )</th>
<th>1% ( = 0.01 )</th>
<th>Capacity (Erlangs) for GOS = 0.005</th>
<th>Capacity (Erlangs) for GOS = 0.002</th>
<th>Capacity (Erlangs) for GOS = 0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.153</td>
<td>0.105</td>
<td>0.065</td>
<td>0.046</td>
</tr>
<tr>
<td>4</td>
<td>0.869</td>
<td>0.701</td>
<td>0.535</td>
<td>0.439</td>
</tr>
<tr>
<td>5</td>
<td>1.36</td>
<td>1.13</td>
<td>0.900</td>
<td>0.762</td>
</tr>
<tr>
<td><strong>10</strong></td>
<td><strong>4.46</strong></td>
<td><strong>3.96</strong></td>
<td><strong>3.43</strong></td>
<td><strong>3.09</strong></td>
</tr>
<tr>
<td><strong>20</strong></td>
<td><strong>12.0</strong></td>
<td><strong>11.1</strong></td>
<td><strong>10.1</strong></td>
<td><strong>9.41</strong></td>
</tr>
<tr>
<td>24</td>
<td>15.3</td>
<td>14.2</td>
<td>13.0</td>
<td>12.2</td>
</tr>
<tr>
<td>40</td>
<td>29.0</td>
<td>27.3</td>
<td>25.7</td>
<td>24.5</td>
</tr>
<tr>
<td>70</td>
<td>56.1</td>
<td>53.7</td>
<td>51.0</td>
<td>49.2</td>
</tr>
<tr>
<td>100</td>
<td>84.1</td>
<td>80.9</td>
<td>77.4</td>
<td>75.2</td>
</tr>
</tbody>
</table>

[Rappaport, 2002, Table 3.4]
Summary of Chapter 2: Big Picture

\[ S = \text{total \# available duplex radio channels for the system} \]

Frequency reuse with \textit{cluster size} \( N \)

\[
C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}
\]

\[
\frac{S}{I} \approx \frac{kR^{-\gamma}}{K \times (kD^{-\gamma})} = \frac{1}{K} \left( \frac{D}{R} \right)^{\gamma} = \frac{1}{K} \left( \sqrt{3}N \right)^{\gamma}
\]

Path loss exponent

“Capacity”

\( m = \# \) channels allocated to each cell.

Omni-directional: \( K = 6 \)
120° Sectoring: \( K = 2 \)
60° Sectoring: \( K = 1 \)

Trunking

\( m = \# \) trunked channels

\( \lambda = \text{Average \# call attempts/requests per unit time} \)

Call blocking probability

\[
P_b = \sum_{i=0}^{m} \frac{A^i}{m! \cdot i!}
\]

\( A = \text{traffic intensity or load [Erlangs]} = \frac{\lambda}{\mu} \)

Erlang-B formula

\[
\frac{1}{\mu} = H = \text{Average call length}
\]
Example 3: System Design (1)

- 20 MHz of total spectrum.
- Each simplex channel has 25 kHz RF bandwidth.
- The number of duplex channels:
  \[ S = \frac{20 \times 10^6}{2 \times 25 \times 10^3} = 400 \text{ channels} \]
- \( \gamma = 4 \)
- Design requirements:
  - SIR \( \geq 15 \) dB
  - \( P_b \leq 5\% \)
- Goal: Maximize the number of users that can be supported by the system.
- Question:
  - \( N = ? \)
  - Should we use sectoring?
Example 3 (2)

- SIR $\geq 15$ dB

\[
\text{SIR} \approx \frac{1}{K} \left( \sqrt{3N} \right)^{\gamma}
\]

K = 1 $\rightarrow$ N = 3
K = 2 $\rightarrow$ N = 3
K = 6 $\rightarrow$ N = 7
Example 3 (3)

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<td>K</td>
<td>6</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>N</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>SIR [dB]</td>
<td>18.7</td>
<td>16.1</td>
<td>19.1</td>
</tr>
<tr>
<td>#channels/cell</td>
<td>$\frac{400}{7}$ = 57</td>
<td>$\frac{400}{3}$ = 133</td>
<td>$\frac{400}{3}$ = 133</td>
</tr>
<tr>
<td>#sectors/cell</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>m = #channels/sector</td>
<td>57</td>
<td>$\left\lfloor \frac{400}{3} \right\rfloor /3 = 44$</td>
<td>$\left\lfloor \frac{400}{3} /6 \right\rfloor = 22$</td>
</tr>
<tr>
<td>A [Erlangs]/sector</td>
<td>51.55</td>
<td>38.56</td>
<td>17.13</td>
</tr>
<tr>
<td>A [Erlangs]/cell</td>
<td>51.55</td>
<td>$38.56 \times 3 = 115.68$</td>
<td>$17.13 \times 6 = 102.78$</td>
</tr>
<tr>
<td>#users/cell</td>
<td>18558</td>
<td>41645</td>
<td>37001</td>
</tr>
</tbody>
</table>

Assume that each user makes 2 calls/day and 2 min/call on average → $1/360$ Erlangs.

Conclusion: With $\gamma = 4$, $\text{SIR} \geq 15 \text{ dB}$, and $P_b \leq 5\%$, $120^\circ$ sectoring with cluster size $N = 3$ should be used.
Example 3 (4): Remarks

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<td>⌊400/7 /6⌋ = 9</td>
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<td>A [Erlangs]/sector</td>
<td>51.55</td>
<td>14.31</td>
<td>5.37</td>
</tr>
<tr>
<td>A [Erlangs]/cell</td>
<td>51.55</td>
<td>14.31×3 = 42.94</td>
<td>5.37×6 = 32.22</td>
</tr>
</tbody>
</table>

For the same $N$, we see that 120° sectoring and 60° sectoring give much better SIR. However, sectoring reduces the trunking efficiency and therefore suffer reduced value of $A$. 
### Idea:
The values of SIR are too high for the cases of 120° sectoring and 60° sectoring. We can further reduce the cluster size. This increases the number of channels per cell and hence per sector.

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