2.4 Traffic Handling Capacity
and Erlang B Formula
Capacity Concept: A Revisit

- Q: If I have $m$ channels per cell, is it true that my cell can support only $m$ users?
- A: Yes and No
- Let’s try one example.
- How often do you make a call?
  - 3 calls a day, on average.
- How long is the call?
  - 10 mins (per call), on average.
- So, one person uses
Capacity Concept: A Revisit

- If we can “give” the time that “User 1” is idle to other users,
  - then one channel can support users!!

- True?
New Concepts for a New Look at Capacity

- We can let more than one user share a channel by using it at different times.
- **Blocked call** happens if a user requests to make a call when all the channels are occupied by other users.
- **Probability of (call) blocking: \( P_b \)**
  - The likelihood that a call is blocked because there is no available channel.
  - 1%, 2%, 5%
- In which case, the number of users that a cell can support can exceed \( S/N \).
  - How much larger depends strongly on the value of \( P_b \) that can be tolerated.
Trunking

- Allow a large number \( n \) of users to **share** the relatively small number \( m \) of channels in a cell (or a sector) by providing access to each user, **on demand**, from a **pool** of available channels.

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<th>60° Sectoring</th>
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<tbody>
<tr>
<td>#sectors/cell</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>#channels/sector</td>
<td>( m = \lfloor S/N \rfloor )</td>
<td>( m = \left\lfloor \frac{S}{N}/3 \right\rfloor )</td>
<td>( m = \left\lfloor \frac{S}{N}/6 \right\rfloor )</td>
</tr>
</tbody>
</table>

- Exploit the **statistical behavior** of users.

- Each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.
Common Terms (1)

- **Traffic Intensity**: Measure of channel time utilization (traffic load / amount of traffic), which is the average channel occupancy measured in **Erlangs**.
  - Dimensionless
  - Denoted by $A$.

- **Holding Time**: Average duration of a typical call.
  - Denoted by $H = 1/\mu$.

- **Request Rate**: The average number of call requests per unit time. Denoted by $\lambda$.

- Use $A_u$ and $\lambda_u$ to denote the corresponding quantities for one user.

- Note that $A = nA_u$ and $\lambda = n\lambda_u$ where $n$ is the number of users supported by the pool (trunked channels) under consideration.
Common Terms (2)

- **Blocked Call**: Call which cannot be completed at time of request, due to congestion.
  - Also referred to as a *lost call*.

- **Grade of Service (GOS)**: A measure of congestion which is specified as the probability of a call being blocked (for Erlang B).
  - The AMPS cellular system is designed for a GOS of 2% blocking.
  - This implies that the channel allocations for cell sites are designed so that, on average, 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.
Erlang B Formula

\[ P_b = \frac{A^m}{\sum_{i=0}^{m} \frac{A^i}{i!}}. \]

\( m = \text{Number of trunked channels} \)

\( A = \text{traffic intensity or load [Erlangs]} \)

\[ = \frac{\lambda}{\mu} \]

\( \lambda = \text{Average # call attempts/requests per unit time} \)

\[ \frac{1}{\mu} = H = \text{Average call length} \]

In MATLAB, use erlangb(m,A)

We use the MATLAB code from http://infohost.nmt.edu/~borchers/erlang.html to evaluate the Erlang B formula.
M/M/m/m Assumption

- **Blocked calls cleared**
  - No queuing for call requests.
  - For every user who requests service, there is no setup time and the user is given immediate access to a channel if one is available.
  - If no channels are available, the requesting user is blocked without access and is free to try again later.

- **Calls arrive as determined by a Poisson process.**
  - There are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time.
  - There are an infinite number of users (with finite overall request rate).
    - The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.

- **The duration of the time that a user occupies a channel is exponentially distributed**, so that longer calls are less likely to occur.

- There are $m$ channels available in the trunking pool.
  - For us, $m = \text{the number of channels for a cell (C) or for a sector}$
Erlang B Formula and Chart

Number of Trunked Channels (m)

Traffic Intensity in Erlangs (A)

\[ P_b = \frac{A^m}{m!} \sum_{i=0}^{m} \frac{A^i}{i!} \]

(log-log plot)
Example 1

- How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?
  (a) 5
  (b) 10

- Assume each user generates $A_u = 0.1$ Erlangs of traffic.
Example 1a

Number of Trunked Channels (m)

Traffic Intensity in Erlangs (A)

\( A \approx 1 \Rightarrow n \approx 10 \) users
Example 1b

Number of Trunked Channel (m)

Traffic Intensity in Erlangs (A)

$A \approx 4 \Rightarrow n \approx 40$ users

\[
\text{erlangb}(10, 3.5) = 0.0023 \\
\text{erlangb}(10, 3.8) = 0.0039 \\
\text{erlangb}(10, 3.9) = 0.0046 \\
\text{erlangb}(10, 3.95) = 0.0049 \\
\text{erlangb}(10, 3.96) = 0.0050 \\
\text{erlangb}(10, 3.97) = 0.0051 \\
\text{erlangb}(10, 4) = 0.0053
\]
Example 2.1

- Consider a cellular system in which
  - an average call lasts two minutes
  - the probability of blocking is to be no more than 1%.
- If there are a total of 399 traffic channels for a seven-cell reuse system, there will be 57 traffic channels per cell.
- From the Erlang B formula, can handle 44.2 Erlangs or 1326 calls per hour.

[Rappaport, 2002, Ex 3.9, p 92]
**Example 2.1: Erlang B**

<table>
<thead>
<tr>
<th>Traffic Intensity in Erlangs (A)</th>
<th>Number of Trunked Channels (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>erlangb(57, 44) = 0.0094</td>
<td></td>
</tr>
<tr>
<td>erlangb(57, 44.2) = 0.0099</td>
<td></td>
</tr>
<tr>
<td>erlangb(57, 44.3) = 0.0102</td>
<td></td>
</tr>
<tr>
<td>erlangb(57, 44.5) = 0.0109</td>
<td></td>
</tr>
<tr>
<td>erlangb(57, 45) = 0.0125</td>
<td></td>
</tr>
</tbody>
</table>

![Graph showing Erlang B traffic intensity]
Example 2.2

- Now employing 120° sectoring, there are only $m = \frac{57}{3} = 19$ channels per sector.
- For the same probability of blocking and average call length, each sector can handle 11.2 Erlangs or 336 calls per hour.
- Since each cell consists of three sectors, this provides a cell capacity of $3 \times 336 = \text{1008 calls per hour}$, which amounts to a 24% decrease when compared to the unsectored case.
- Thus, sectoring decreases the trunking efficiency while improving the SIR for each user in the system.

[Rappaport, 2002, Ex 3.9, p 92]
Example 2.2: Erlang B

Number of Trunked Channel (m)

Traffic Intensity in Erlangs (A)

- $\text{erlangb}(19, 11) = 0.0085$
- $\text{erlangb}(19, 11.2) = 0.0098$
- $\text{erlangb}(19, 11.3) = 0.0105$
- $\text{erlangb}(19, 11.5) = 0.0120$
- $\text{erlangb}(19, 12) = 0.0165$
## Erlang B Trunking Efficiency

<table>
<thead>
<tr>
<th>Number of Channels $m$</th>
<th>Capacity (Erlangs) for GOS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$0.01$</td>
</tr>
<tr>
<td>2</td>
<td>0.153</td>
</tr>
<tr>
<td>4</td>
<td>0.869</td>
</tr>
<tr>
<td>5</td>
<td>1.36</td>
</tr>
<tr>
<td>10</td>
<td>4.46</td>
</tr>
<tr>
<td>20</td>
<td>12.0</td>
</tr>
<tr>
<td>24</td>
<td>15.3</td>
</tr>
<tr>
<td>40</td>
<td>29.0</td>
</tr>
<tr>
<td>70</td>
<td>56.1</td>
</tr>
<tr>
<td>100</td>
<td>84.1</td>
</tr>
</tbody>
</table>

[Rappaport, 2002, Table 3.4]
Summary of Chapter 2: Big Picture

\[ S = \text{total \# available duplex radio channels for the system} \]

**“Capacity”**

\[ C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N} \]

Frequency reuse with **cluster size** \( N \)

\[ \frac{S}{I} \approx \frac{kR^{-\gamma}}{K \times (kD^{-\gamma})} = \frac{1}{K} \left( \frac{D}{R} \right)^{\gamma} = \frac{1}{K} \left( \sqrt{3N} \right)^{\gamma} \]

Path loss exponent

\( m = \# \text{ channels allocated to each cell.} \)

**Trunking**

\( m = \# \text{ trunked channels} \)

\( \lambda = \text{Average \# call attempts/requests per unit time} \)

\( A = \text{traffic intensity or load [Erlangs]} = \frac{\lambda}{\mu} \)

Erlang-B formula

\[ \frac{1}{\mu} = H = \text{Average call length} \]
Example 3: System Design (1)

- 20 MHz of total spectrum.
- Each simplex channel has 25 kHz RF bandwidth.
- The number of duplex channels:
  \[
  S = \frac{20 \times 10^6}{2 \times 25 \times 10^3} = 400 \text{ channels}
  \]
- \( \gamma = 4 \)
- Design requirements:
  - SIR \( \geq 15 \text{ dB} \)
  - \( P_b \leq 5\% \)
- Goal: Maximize the number of users that can be supported by the system.
- Question:
  - \( N = ? \)
  - Should we use sectoring?
Example 3 (2)

- SIR $\geq 15$ dB

$$\text{SIR} \approx \frac{1}{K} \left( \sqrt{3N} \right)^y$$

K = 1 $\rightarrow$ N = 3
K = 2 $\rightarrow$ N = 3
K = 6 $\rightarrow$ N = 7
Example 3 (3)

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<tr>
<td>K</td>
<td>6</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>N</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>SIR [dB]</td>
<td>18.7</td>
<td>16.1</td>
<td>19.1</td>
</tr>
<tr>
<td>#channels/cell</td>
<td>[400/7] = 57</td>
<td>[400/3] = 133</td>
<td>[400/3] = 133</td>
</tr>
<tr>
<td>#sectors/cell</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>m = #channels/sector</td>
<td>57</td>
<td>(\left\lfloor \frac{400}{3} \right\rfloor = 44)</td>
<td>(\left\lfloor \frac{400}{3} /6 \right\rfloor = 22)</td>
</tr>
<tr>
<td>A [Erlangs]/sector</td>
<td>51.55</td>
<td>38.56</td>
<td>17.13</td>
</tr>
<tr>
<td>A [Erlangs]/cell</td>
<td>51.55</td>
<td>38.56×3 = 115.68</td>
<td>17.13×6 = 102.78</td>
</tr>
<tr>
<td>#users/cell</td>
<td>18558</td>
<td>41645</td>
<td>37001</td>
</tr>
</tbody>
</table>

Assume that each user makes 2 calls/day and 2 min/call on average → \(1/360\) Erlangs.

Conclusion: With \(\gamma = 4\), \(\text{SIR} \geq 15\,\text{dB}\), and \(\text{Pb} \leq 5\%\),
120° sectoring with cluster size \(N = 3\) should be used.
Example 3 (4): Remarks

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<td>m = #channels/sector</td>
<td>57</td>
<td>$\left\lfloor \frac{400}{7}/3 \right\rfloor = 19$</td>
<td>$\left\lfloor \frac{400}{7}/6 \right\rfloor = 9$</td>
</tr>
<tr>
<td>A [Erlangs]/sector</td>
<td>51.55</td>
<td>14.31</td>
<td>5.37</td>
</tr>
<tr>
<td>A [Erlangs]/cell</td>
<td>51.55</td>
<td>$14.31 \times 3 = 42.94$</td>
<td>$5.37 \times 6 = 32.22$</td>
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For the same $N$, we see that 120° sectoring and 60° sectoring give much better SIR. However, sectoring reduces the trunking efficiency and therefore suffer reduced value of A.
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Idea: The values of SIR are too high for the cases of 120° sectoring and 60° sectoring. We can further reduce the cluster size. This increases the number of channels per cell and hence per sector.