2.2 Co-Channel Interference
Co-Channel Interference

- Frequency reuse $\rightarrow$ co-channel interference
- Consider only nearby interferers.
  - Power decreases rapidly as the distance increases.
- In a fully equipped hexagonal-shaped cellular system, there are always $K = 6$ cochannel interfering cells in the first tier.
Three Measures of Signal Quality

- For **noise-limited** systems, \( \text{SNR} = \frac{P_r}{P_{\text{noise}}} \)

- Consider both noise & interference: \( \text{SINR} = \frac{P_r}{P_{\text{interference}} + P_{\text{noise}}} \)

- The best cellular system design places users that share the same channel at a separation distance (as close as possible) where the intercell interference is just below the maximum tolerable level for the required data rate and BER.

- Good cellular system designs are **interference-limited**, meaning that the interference power is much larger than the noise power.

\[ \text{SIR} = \frac{P_r}{P_{\text{interference}}} \]
“Reliable” vs. “tolerable”?

(Why not as far as possible?)

Co-channel cells, must be spaced far enough apart so that interference between users in co-channel cells does not degrade signal quality below tolerable levels.

Subjective tests found that people regard an FM signal using a 30 kHz channel bandwidth to be clear if the signal power is at least sixty times higher than the noise/interference power.

\[10 \log_{10} 60 = 17.78 \approx 18 \text{ dB}\]

We will soon revisit and use these numbers for some more specific calculations.

[Klemens, 2010, p 54]
Review: Simplified Path Loss Model

\[ \frac{P_r}{P_t} = \beta \left( \frac{d_0}{d} \right)^\gamma \]

- \( \beta \) is a unitless constant which depends on the antenna characteristics and the average channel attenuation.
- \( d_0 \) is a reference distance for the antenna far-field:
  - Typically 1-10 m indoors and 10-100 m outdoors.
- \( \gamma \) is the path loss exponent:
  - 2 in free-space model
  - 4 in two-ray model [Goldsmith, 2005, eq. 2.17]

Capture the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

<table>
<thead>
<tr>
<th>Environment</th>
<th>( \gamma ) range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban macrocells</td>
<td>3.7-6.5</td>
</tr>
<tr>
<td>Urban microcells</td>
<td>2.7-3.5</td>
</tr>
<tr>
<td>Office Building (same floor)</td>
<td>1.6-3.5</td>
</tr>
<tr>
<td>Office Building (multiple floors)</td>
<td>2-6</td>
</tr>
<tr>
<td>Store</td>
<td>1.8-2.2</td>
</tr>
<tr>
<td>Factory</td>
<td>1.6-3.3</td>
</tr>
<tr>
<td>Home</td>
<td>3</td>
</tr>
</tbody>
</table>

[Goldsmith, 2005, Table 2.2]
SIR (S/I): Definition/Calculation

- $K = \# \text{ co-channel interfering cells}$
- The **signal-to-interference ratio** (S/I or SIR) for a mobile receiver which monitors a forward channel can be expressed as

\[
\text{SIR} = \frac{P_r}{\sum_{i=1}^{K} P_{\text{interference of the } i^{th} \text{ interferer}}} = \frac{P_r}{\sum_{i} P_i}
\]

- $P_r = \text{the desired signal power from the desired base station}$
- $P_i = \text{the interference power caused by the } i^{th} \text{ interfering co-channel cell base station}$.
- Often called the **carrier-to-interference ratio**: CIR.

[Caution: Not the same as the K used in Section 1.3]

[Rappaport, 2002]
SIR: N = 3
Hexagon

Area = 6 \times 2 \times \left( \frac{1}{2} \times \frac{\sqrt{3}}{2} R \times \frac{1}{2} R \right) = \frac{3\sqrt{3}}{2} R^2 \approx 2.598 R^2
SIR: $N = 3$

- Consider only cells in first tier.
- Worse-case distance

\[
\text{SIR} \approx \frac{\sum_{i} k_i / D_i^\gamma}{\sum_{i} 1 / \left( \frac{D_i}{R} \right)^\gamma} = \frac{1}{\sum_{i} \left( \frac{D_i}{R} \right)^{-\gamma}} = \frac{1}{2 \left( \sqrt{7} \right)^{-\gamma} + 2 \left( \sqrt{13} \right)^{-\gamma} + 2^{-\gamma} + 4^{-\gamma}}
\]

If $N = 19$, will the SIR be better or worse?
SIR: $N = 3$

Observe that the SIR value is smallest when MS is at any of the corners of the hexagonal cell. At such locations, $d = R$ (the cell radius).

$d = \text{distance between MS and BS}$

$$
\text{SIR} \approx \frac{k}{\sum_i D_i^{\gamma}} = \frac{1}{\sum_i \left( \frac{D_i}{d} \right)^{\gamma}} = \frac{1}{\sum_i \left( \frac{D_i}{d} \right)^{-\gamma}}
$$

Centers of cochannel cells when $N = 3$
SIR: $N = 3$ vs. $N = 7$

Centers of cochannel cells when $N = 3$

Centers of cochannel cells when $N = 7$
Approximation

- Consider only first tier.
- Worse-case distance
  \[
  \text{SIR} \approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}}
  \]
- Use the same $D$ for $D_i$
Approximation

- Consider only first tier.
- Worse-case distance

\[
SIR \approx \frac{1}{\sum_i \left( \frac{D_i}{R} \right)^{-\gamma}}
\]

- Use the same \( D \) for \( D_i \)

\[
SIR \approx \frac{1}{\sum_i \left( \frac{D}{R} \right)^{-\gamma}} \approx \frac{1}{K \left( \frac{D}{R} \right)^{-\gamma}} = \frac{1}{K \left( \frac{D}{R} \right)^{\gamma}}
\]

Notice that \( D/R \) is an important quantity!
Center-to-center distance (D)

This distance, $D$, is called reuse distance.

Co-channel reuse ratio

$$Q = \frac{D}{R} = \sqrt{3}N.$$
Q and N

Co-channel reuse ratio

\[ Q = \frac{D}{R} = \sqrt{3N}. \]

<table>
<thead>
<tr>
<th>Cluster Size (N)</th>
<th>Co-channel Reuse Ratio (Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 1, j = 1 )</td>
<td>3</td>
</tr>
<tr>
<td>( i = 1, j = 2 )</td>
<td>7</td>
</tr>
<tr>
<td>( i = 0, j = 3 )</td>
<td>9</td>
</tr>
<tr>
<td>( i = 2, j = 2 )</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>( \sqrt{3N} )</td>
</tr>
</tbody>
</table>
Approximation: Crude formula

As the cell cluster size (N) increases, the spacing (D) between interfering cells increases, reducing the interference.
Summary: Quantity vs. Quality

\[ S = \text{total \# available duplex radio channels for the system} \]

“Capacity” \[ C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N} \]

Frequency reuse with \textit{cluster size} \( N \)

Path loss exponent

\[ \text{SIR} \approx \frac{1}{K} \left( \frac{C}{\sqrt{3N}} \right)^\gamma \]

\( m = \# \text{ channels allocated to each cell.} \)
SIR: N = 7

Better approximation...

\[
SIR \approx \frac{R^{-\gamma}}{2(D - R)^{-\gamma} + 2(D + R)^{-\gamma} + 2D^{-\gamma}}
\]

\[
= \frac{1}{2(Q - 1)^{-\gamma} + 2(Q + 1)^{-\gamma} + 2Q^{-\gamma}}
\]

Again, \( Q = \frac{D}{R} = \sqrt{3N} \).
Comparison

\[
\text{SIR} \approx \frac{1}{\sum_{i=1}^{6} \left( \frac{D_i}{R} \right)^{-\gamma}}
\]

\[
\text{SIR} \approx \frac{1}{6} Q^\gamma
\]

\[
\text{SIR} \approx \frac{1}{2(Q-1)^{-\gamma} + 2(Q+1)^{-\gamma} + 2Q^{-\gamma}}
\]

\[
Q = \frac{D}{R}
\]
SIR Threshold

- The SIR should be greater than a specified threshold for proper signal operation.
- In the 1G AMPS system, designed for voice calls, the threshold for acceptable voice quality is SIR equal to 18 dB.
- For the 2G digital AMPS system (D-AMPS or IS-54/136), a threshold of 14 dB is deemed suitable.
- For the GSM system, a range of 7–12 dB, depending on the study done, is suggested as the appropriate threshold.
- The probability of error in a digital system depends on the choice of this threshold as well.