Name ________________________ ID3 _ _ _

ECS 452: Digital Communication Systems **2019/2**

HW 8 — Due: Not Due

Lecturer: Prapun Suksompong, Ph.D.

Problem 1. Consider a convolutional encoder whose trellis diagram is given in Figure 8.1.

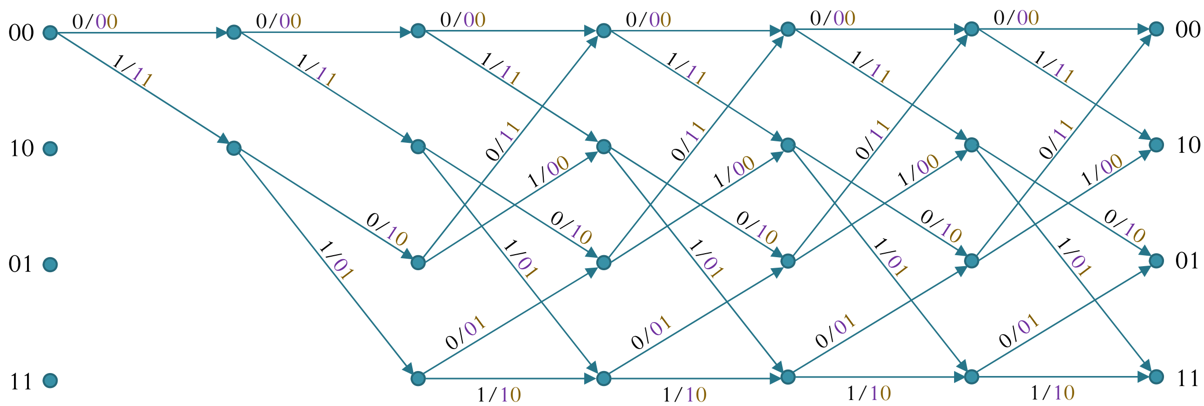

Figure 8.1: State diagram for a convolutional encoder

(a) Find the code rate

(b) Suppose the data bits (message) are $\underline{\mathbf{b}} = [0100101]$. Find the corresponding codeword $\underline{\mathbf{x}}$.

(c) Find the data vector $\underline{\mathbf{b}}$ which gives the codeword $\underline{\mathbf{x}} = [001110111110]$.

(d) Suppose that we observe $\underline{\mathbf{y}}$ = [001110000101] at the input of the minimum distance decoder. Explain why we can easily find the decoded codeword $\hat{\underline{\mathbf{x}}}$ and the decoded message $\hat{\underline{\mathbf{b}}}$ without applying the Viterbi algorithm.

(e) Suppose that we observe $\underline{\mathbf{y}}$ = [010101111110] at the input of the minimum distance decoder. Use Viterbi algorithm to find the decoded codeword $\hat{\underline{\mathbf{x}}}$ and the decoded message $\hat{\underline{\mathbf{b}}}$. Show your work on Figure 8.2 below.

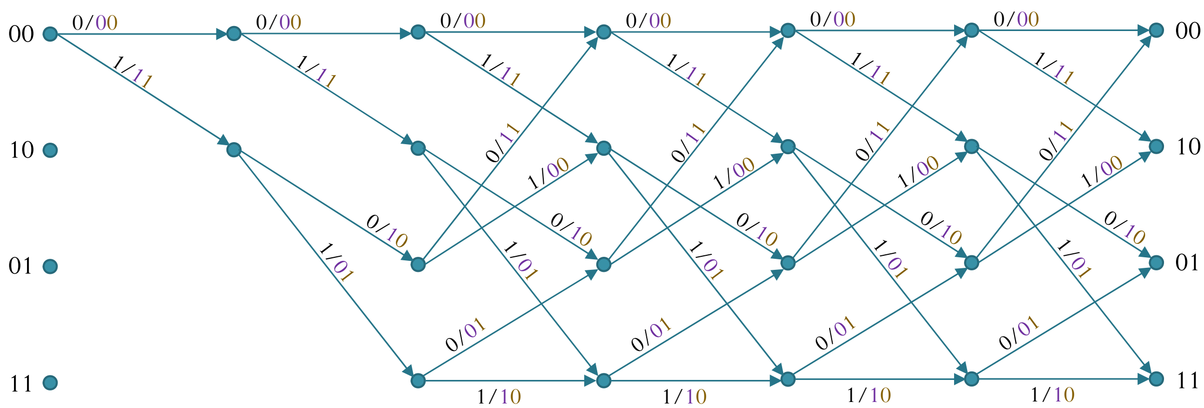

Figure 8.2: State diagram for a convolutional encoder

Make sure that all the running (cumulative) path metric are shown and the discarded branches are indicated at every steps.

Problem 2. Consider four vectors:

$$\mathbf{v}^{(1)} = \begin{pmatrix} +1 \\ -1 \\ +1 \\ 0 \\ -1 \end{pmatrix}, \mathbf{v}^{(2)} = \begin{pmatrix} +1 \\ +1 \\ 0 \\ +1 \\ 0 \end{pmatrix}, \mathbf{v}^{(3)} = \begin{pmatrix} +2 \\ 0 \\ +1 \\ +1 \\ -1 \end{pmatrix}, \text{ and } \mathbf{v}^{(4)} = \begin{pmatrix} +3 \\ +1 \\ +1 \\ +2 \\ -1 \end{pmatrix}.$$

Now, consider two vectors:

$$\mathbf{e}^{(1)} = \frac{1}{2}\begin{pmatrix} +1 \\ -1 \\ +1 \\ 0 \\ -1 \end{pmatrix}, \text{ and } \mathbf{e}^{(2)} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

(a) Show that the two vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are orthonormal.

(b) Find the corresponding vectors $\mathbf{c}^{(1)}$, $\mathbf{c}^{(2)}$, $\mathbf{c}^{(3)}$, and $\mathbf{c}^{(4)}$ that represent $\mathbf{v}^{(1)}$, $\mathbf{v}^{(2)}$, $\mathbf{v}^{(3)}$, and $\mathbf{v}^{(4)}$ in the new coordinate system defined by vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$.

Problem 3. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 8.4. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

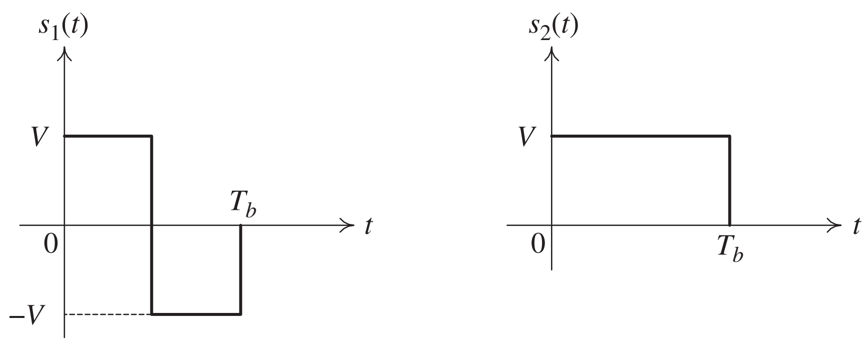

Figure 8.4: Signal set for Problem 3

(a) Find the energy in each signal.

(b) Find the two vectors that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space with respect to the following orthonormal vectors:

$$\phi_1(t) = \begin{cases} \frac{1}{\sqrt{T_b}}, & 0 \le t \le \frac{T_b}{2}, \\ -\frac{1}{\sqrt{T_b}}, & \frac{T_b}{2} < t < T_b, \\ 0, & \text{otherwise} \end{cases} \qquad \phi_2(t) = \begin{cases} \frac{1}{\sqrt{T_b}}, & 0 \le t \le T_b, \\ 0, & \text{otherwise} \end{cases}$$

Also draw the corresponding constellation.

Problem 4. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 8.5. Note that V, α and T_b are some positive constants.

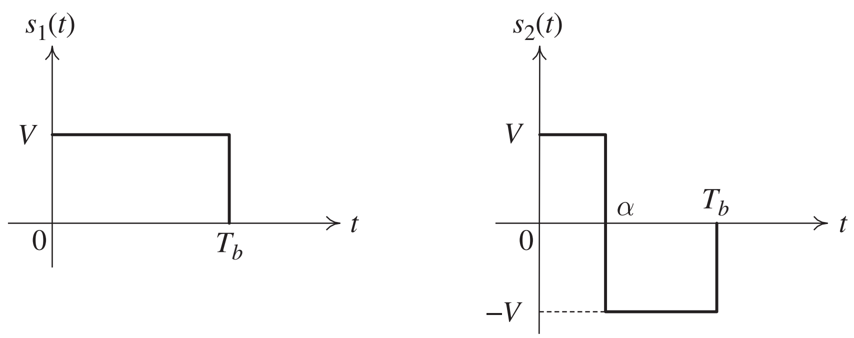

Figure 8.5: Signal set for Problem 4

(a) Find the energy in each signal.

(b) Find $\langle s_1(t), s_2(t)\rangle$.

(c) Consider two orthonormal vectors:

$$\phi_1(t) = \begin{cases} \frac{1}{\sqrt{T_b}}, & 0 < t < T_b, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad \phi_2(t) = \frac{1}{\sqrt{\alpha\left(1-\frac{\alpha}{T_b}\right)}} \times \begin{cases} 1-\frac{\alpha}{T_b} & 0 < t \le \alpha, \\ -\frac{\alpha}{T_b}, & \alpha < t < T_b, \\ 0, & \text{otherwise} \end{cases}$$

(i) Check that they are orthonormal.

(ii) Find the two vectors $\mathbf{s}^{(1)}$ and $\mathbf{s}^{(2)}$ that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space with respect to the given orthonormal vectors.

(iii) Draw the corresponding constellation when $\alpha = \frac{T_b}{4}$.

(iv) Draw $\mathbf{s}^{(2)}$ when $\alpha = \frac{k}{10}T_b$ where $k = 1, 2, \ldots, 9$.

Extra Question

Here is an optional question for those who want more practice.

Problem 5. Consider a convolutional code generated by the encoder shown in Figure 8.3. Suppose that we observe $\underline{\mathbf{y}} = [110111000110]$ at the input of the minimum distance decoder. Use Viterbi algorithm to find the decoded codeword $\hat{\underline{\mathbf{x}}}$ and the decoded message $\hat{\underline{\mathbf{b}}}$. Caution: The trellis diagram is not the same as the one used in Problem 1.

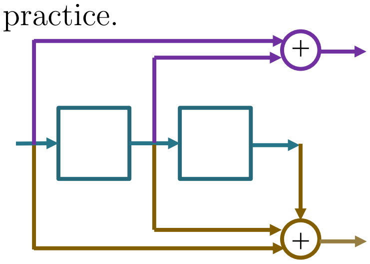

Figure 8.3: Encoder for Problem 5

Problem 6. Suppose $s_1(t) = \text{sinc}(5t)$ and $s_2(t) = \text{sinc}(7t)$. Note that in this class, we define $\text{sinc}(x) = \frac{\sin x}{x}$. Find

(a) E_{s_1},

(b) E_{s_2}, and

(c) $\langle s_1(t), s_2(t) \rangle$.

Hint: Use Parseval's theorem to evaluate the above quantities in the frequency domain. (Review: See 2.43 on p. 22 and Example 2.44 on p. 23 of ECS332 2019 lecture notes and Problem 6 in ECS332 2019 HW4.)

Problem 7. Prove the following facts with the help of Fourier transform.
(Hint: inner product in the frequency domain, Parseval's theorem)

(a) The energy of $p(t) = g(t)\cos(2\pi f_c t + \phi)$ is $E_g/2$.

(b) $g(t)\cos(2\pi f_c t)$ and $-g(t)\sin(2\pi f_c t)$ are orthogonal.

Is there any condition(s) on $g(t)$ that we need to assume?