

ECS 452: In-Class Exercise # 1

Instructions

1. Separate into groups of no more than three students each.
2. [ENRE] Explanation is not required for this exercise.
3. **Do not panic.**

Date: 17 / 1 / 2020			
Name			ID <small>(last 3 digits)</small>

1. Consider two codes (for source coding) below. The left column is for Code A. The right column is for Code B. The first row defines these codes via their codebooks.

<p>Codebook for Code A</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>x</td> <td>a</td> <td>e</td> <td>ℓ</td> <td>n</td> <td>r</td> </tr> <tr> <td>$c(x)$</td> <td>1</td> <td>00</td> <td>010</td> <td>0110</td> <td>0111</td> </tr> </table>	x	a	e	ℓ	n	r	$c(x)$	1	00	010	0110	0111	<p>Codebook for Code B</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>x</td> <td>a</td> <td>e</td> <td>ℓ</td> <td>n</td> <td>r</td> </tr> <tr> <td>$c(x)$</td> <td>000</td> <td>100</td> <td>10</td> <td>01</td> <td>11</td> </tr> </table>	x	a	e	ℓ	n	r	$c(x)$	000	100	10	01	11
x	a	e	ℓ	n	r																				
$c(x)$	1	00	010	0110	0111																				
x	a	e	ℓ	n	r																				
$c(x)$	000	100	10	01	11																				
The source alphabet for Code A is	The source alphabet for Code B is																								
The code alphabet for Code A is	The code alphabet for Code B is																								
Use code A to encode the source string “real”	Use code B to encode the source string “real”																								
Is Code A nonsingular?	Is Code B nonsingular?																								
The string 01000101110110 comes from encoding a source string by Code A. Decode it.	The string 0001110001000 comes from encoding a source string by Code B. Decode it.																								

ECS 452: In-Class Exercise # 2

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 21 / 1 / 2020			
Name			ID <small>(last 3 digits)</small>

1. Consider a DMS whose source alphabet is $\{a, e, \ell, n, r\}$.

The probabilities for these five symbols are shown in the table below:

x	a	e	ℓ	n	r
$p(x)$	0.1	0.2	0.2	0.2	0.3

Consider two codes (for source coding) below.

The left column is for Code A. The right column is for Code B.

The first row defines these codes via their codebooks.

<p>Codebook for Code A</p> <table border="1" style="border-collapse: collapse; text-align: center; margin-bottom: 10px;"> <tr> <td>x</td> <td>a</td> <td>e</td> <td>ℓ</td> <td>n</td> <td>r</td> </tr> <tr> <td>$c(x)$</td> <td>1</td> <td>00</td> <td>010</td> <td>0110</td> <td>0111</td> </tr> </table> <p>Suppose the DMS above is encoded by Code A. Find the expected codeword length.</p>	x	a	e	ℓ	n	r	$c(x)$	1	00	010	0110	0111	<p>Codebook for Code B</p> <table border="1" style="border-collapse: collapse; text-align: center; margin-bottom: 10px;"> <tr> <td>x</td> <td>a</td> <td>e</td> <td>ℓ</td> <td>n</td> <td>r</td> </tr> <tr> <td>$c(x)$</td> <td>000</td> <td>100</td> <td>10</td> <td>01</td> <td>11</td> </tr> </table> <p>Suppose the DMS above is encoded by Code B. Find the expected codeword length.</p>	x	a	e	ℓ	n	r	$c(x)$	000	100	10	01	11
x	a	e	ℓ	n	r																				
$c(x)$	1	00	010	0110	0111																				
x	a	e	ℓ	n	r																				
$c(x)$	000	100	10	01	11																				
<p>Draw the code tree for Code A.</p>	<p>Draw the code tree for Code B.</p>																								
<p>Is Code A prefix-free?</p>	<p>Is Code B prefix-free?</p>																								

ECS 452: In-Class Exercise # 3

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 24 / 1 / 2020			
Name			ID <small>(last 3 digits)</small>

1. Consider a random variable X which has five possible values. Their probabilities are shown in the table below.

x	$p_X(x)$	$c(x)$	$\ell(x)$	$\log_2 \frac{1}{p_X(x)}$
a	0.2	110		
e	0.1			
ℓ	0.1			
n	0.3			
r	0.3			

- a. Find a binary Huffman code (without extension) for this random variable. Put the values of the codewords and the codeword lengths in the table above. Note that the codeword for the source symbol “ ℓ ” is required to be 110. (So, you do not have complete freedom on assigning 0 and 1 to the edges.)
- b. Find the expected codeword length (per source symbol) when your Huffman coding in part (a) is used (without extension).

c. Calculate $\sum_x \left(p_X(x) \left(\log_2 \frac{1}{p_X(x)} \right) \right)$

(Note that $\log_2 c = \frac{\log_{10} c}{\log_{10} 2} = \frac{\ln c}{\ln 2}$.)

ECS 452: In-Class Exercise # 4

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 28 / 1 / 2020			
Name	ID <small>(last 3 digits)</small>		

1. A discrete memoryless source emits three possible messages Yes, No, and OK with probabilities 0.1, 0.1, and 0.8, respectively.
 - a. Find the expected codeword length when a binary Huffman code is constructed without extension.

- b. Find the codeword lengths when binary Huffman code with second-order extension is used to encode this source. Put the values of the corresponding probabilities and the codeword lengths in the table below. (Note that, for brevity, we use Y,N,K to represent Yes, No, and OK, respectively.)

x_1x_2	$p_{X_1, X_2}(x_1, x_2)$	$\ell(x_1, x_2)$
YY		
YN		
YK		
NY		
NN		
NK		
KY		
KN		
KK		

- c. Find L_2 .
(This is the expected codeword length **per source symbol** of the Huffman binary code for the second-order extension of this source.)

ECS 452: In-Class Exercise # 4

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 31 / 1 / 2020			
Name			ID <small>(last 3 digits)</small>

1. In each part below, we consider a random variable X which has five possible values. The probability for each possible value is listed in the provided table. Calculate the corresponding entropy value.

a.

x	a	e	ℓ	n	r
$p(x)$	0.25	0.25	0.125	0.25	0.125

b.

x	a	e	ℓ	n	r
$p(x)$	0.1	0.2	0.2	0.2	0.3

2. [ENRPr] In each row of the following table, compare the quantity in the first column with the one in the third column by writing “>”, “=”, or “<” in the second column. Watch out for approximation error.

(a)	$H(\underline{\mathbf{p}})$ when $\underline{\mathbf{p}} = [0.4, 0.6]$		$H(\underline{\mathbf{p}})$ when $\underline{\mathbf{p}} = [0.3, 0.7]$
(b)	$H(\underline{\mathbf{p}})$ when $\underline{\mathbf{p}} = [0.4, 0.6]$		$H(\underline{\mathbf{p}})$ when $\underline{\mathbf{p}} = [0.1, 0.3, 0.6]$
(c)	$H(\underline{\mathbf{p}})$ when $\underline{\mathbf{p}} = [0.1, 0.2, 0.3, 0.4]$		$H(\underline{\mathbf{p}})$ when $\underline{\mathbf{p}} = [0.1, 0.3, 0.2, 0.4]$

ECS 452: In-Class Exercise # 6

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. [ENRE] Explanation is not required for this exercise.
3. **Do not panic.**

Date: 4 / 2 / 2020			
Name	ID <small>(last 3 digits)</small>		

1. Consider a DMC whose samples of input and output are provided below

x: 0 0 0 1 0 1 0 1 0 0 1 0 0 1 1
 y: 0 0 0 1 0 1 0 0 1 0 0 1 0 0 1

Estimate the following quantities:

- | | |
|---|---|
| <p>a. \mathcal{X}</p> <p>b. \mathcal{Y}</p> <p>c. $P[X = 0]$</p> <p>d. $p_Y(0)$</p> <p>e. $\underline{\mathbf{p}}$</p> <p>f. $\underline{\mathbf{q}}$</p> <p>g. $P[Y = 0 X = 0]$</p> | <p>h. $p_{Y X}(1 0)$</p> <p>i. $Q(0 1)$</p> <p>j. Matrix \mathbf{Q}</p> <p>k. $P[X = 0, Y = 0]$</p> |
|---|---|

ECS 452: In-Class Exercise # 7

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. [ENRE] Explanation is not required for this exercise.
3. [WASP] Write your answer(s) in the corresponding space(s) provided.
4. Do not panic.

Date: 7 / 2 / 2020			
Name			ID <small>(last 3 digits)</small>

1. Consider a binary channel whose $Q(0|0) = 0.7$ and $Q(0|1) = 0.3$. Suppose $P[X = 1] = 0.4$. Find the channel matrix \mathbf{Q} , the output probability vector \mathbf{q} , and the joint pmf matrix \mathbf{P} .

Q matrix	P matrix
	\mathbf{q}

2. Consider a DMC whose channel matrix \mathbf{Q} is given below. Suppose the input probability vector is $\mathbf{p} = [0.4, 0.3, 0.2, 0.1]$. Calculate the missing values in the \mathbf{Q} matrix. Then, find the output probability vector \mathbf{q} and the joint pmf matrix \mathbf{P} .

Q matrix

$x \backslash y$	1	2	3	4
1		0.2	0.3	0.3
2	0.1		0.2	0.3
3	0.3	0.1		0.5
4	0.1	0.1	0.7	

P matrix
\mathbf{q}

ECS 452: In-Class Exercise # 8

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. [ENRE] Explanation is not required for this exercise.
3. [WASP] Write your answer(s) in the corresponding space(s) provided.
4. Do not panic.

Date: 14 / 2 / 2020			
Name			ID <small>(last 3 digits)</small>

1. Consider a DMC whose transition matrix \mathbf{Q} and joint pmf matrix \mathbf{P} are given below.

$$\mathbf{Q} = \begin{array}{c|cccc} x \backslash y & 1 & 2 & 3 & 4 \\ \hline 1 & 0.2 & 0.2 & 0.3 & 0.3 \\ 2 & 0.1 & 0.4 & 0.2 & 0.3 \\ 3 & 0.3 & 0.1 & 0.1 & 0.5 \\ 4 & 0.1 & 0.1 & 0.7 & 0.1 \end{array}$$

$$\mathbf{P} = \begin{array}{c|cccc} x \backslash y & 1 & 2 & 3 & 4 \\ \hline 1 & 0.08 & 0.08 & 0.12 & 0.12 \\ 2 & 0.03 & 0.12 & 0.06 & 0.09 \\ 3 & 0.06 & 0.02 & 0.02 & 0.10 \\ 4 & 0.01 & 0.01 & 0.07 & 0.01 \end{array}$$

We will consider four decoders:

- (a) The naïve decoder.
- (b) A DIY decoder defined by $\hat{x}_{\text{DIY}}(y) = 5 - y$.
- (c) The MAP decoder.
- (d) The ML decoder.

Find the decoding table for each of these decoders. Also, find their probabilities of decoding error. Put your answers in the table below.

y	$\hat{x}_{\text{naïve}}(y)$	$\hat{x}_{\text{DIY}}(y)$	$\hat{x}_{\text{MAP}}(y)$	$\hat{x}_{\text{ML}}(y)$
1				
2				
3				
4				
$P(\mathcal{E})$				

ECS 452: Exercise # 9

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. [ENRE] Explanation is not required for this exercise.
3. [WASP] Write your answer(s) in the corresponding space(s) provided.
4. Do not panic.

Date: 18 / 2 / 2019			
Name	ID <small>(last 3 digits)</small>		

Consider a transmission of equally-likely codewords over a BSC using block encoding.
Codes whose codewords are just “reordering” of another code will be considered the same.

Suppose the crossover probability of the BSC is $p = 0.2$.

Use the provided MATLAB function `PE_minDist` to answer these problems.

1. Suppose a repetition code with $n = 5$ is used. Find the corresponding error probability $P(\mathcal{E})$.

$$P(\mathcal{E}) \approx \underline{\hspace{1cm}}$$

2. Here, as in Problem 1, we will use $k = 1$ and $n = 5$.
 This implies that any code must contain two 5-bit codewords.
 Assume that these two codewords are distinct.

- a. Give an example of a code that performs worse (gives larger $P(\mathcal{E})$) than the repetition code above.

$$\mathcal{C} = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$$

The corresponding $P(\mathcal{E})$ for this code is $\underline{\hspace{1cm}}$.

- b. Give an example of a code that is not a repetition code but performs as good as the repetition code.

$$\mathcal{C} = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$$

3. In this problem, we will use $k = 2$ and $n = 5$.
 This implies that any code must contain four 5-bit codewords.
 Assume that these four codewords are distinct.

- a. Find the error probability $P(\mathcal{E})$ of the following code: $\mathcal{C} = \{00000, 10001, 01110, 11111\}$

$$P(\mathcal{E}) = \underline{\hspace{1cm}}$$

- b. The best value of the error probability $P(\mathcal{E})$ for $k = 2$ and $n = 5$ is 0.2218.

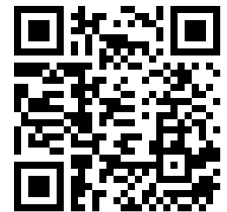
Find at least one code that can achieve this $P(\mathcal{E})$.

$$\mathcal{C} = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$$

Submit your answer of this part at <https://forms.gle/THbSRSqDWRpvg1329>.

You will get a penalty of “-1” if your answer is the same as the answer from another group who submitted before you. (Again, **codes whose codewords are just “reordering” of another code will be considered the same.** For example, $\mathcal{C} = \{10001, 00000, 01110, 11111\}$ is considered the same as the code in part 3a.)

Submitted codes are published at <https://bit.ly/38tLsRe>.



ECS 452: In-Class Exercise # 10

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 21 / 2 / 2019

Name

ID (last 3 digits)

1. Consider two random variables X and Y whose joint pmf matrix is given by

$$\mathbf{P} = \begin{array}{c} x \backslash y \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array} \begin{array}{cc} \begin{array}{c} 1 \\ 2 \end{array} \\ \begin{bmatrix} 1/4 & 0 \\ 1/4 & 1/4 \\ 0 & 1/4 \end{bmatrix} \end{array}$$

Calculate the following quantities.

a. $H(X, Y)$

b. $H(X)$

c. $H(Y)$

d. $H(Y|X)$

e. \mathbf{Q} matrix

f. $H(Y|X = 2)$

ECS 452: In-Class Exercise # 11

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. [ENRE] Explanation is not required for this exercise
3. **Do not panic.**

Date: 28 / 2 / 2019			
Name			ID <small>(last 3 digits)</small>

1. For each of the following DMC's probability transition matrices \mathbf{Q} , (i) indicate whether the corresponding DMC is symmetric (Yes or No), (ii) indicate whether the corresponding DMC is weakly symmetric (Yes or No), (iii) evaluate the corresponding capacity value (your answer should be of the form X.XXXX).

\mathbf{Q}	Symmetric Channel?	Weakly Symmetric Channel?	C
$\begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$			
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}$			
$\begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \\ 0.5 & 0.25 & 0.25 \end{bmatrix}$			
$\begin{bmatrix} 0.75 & 0.25 & 0 & 0 \\ 0 & 0.25 & 0 & 0.75 \\ 0 & 0.25 & 0.75 & 0 \end{bmatrix}$			