ECS 452: In-Class Exercise # 8

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **[ENRE] Explanation is not required for this exercise.**
3. **[WASP] Write your answer(s) in the corresponding space(s) provided.**
4. **Do not panic.**

Date: 14 / 2 / 2020			
Name	ID (last 3 digits)		

1. Consider a DMC whose transition matrix **Q** and joint pmf matrix **P** are given below.

$$\mathbf{Q} = \begin{array}{c|cccc} x \backslash y & 1 & 2 & 3 & 4 \\ 1 & 0.2 & 0.2 & 0.3 & 0.3 \\ 2 & 0.1 & 0.4 & 0.2 & 0.3 \\ 3 & 0.3 & 0.1 & 0.1 & 0.5 \\ 4 & 0.1 & 0.1 & 0.7 & 0.1 \end{array} \qquad \mathbf{P} = \begin{array}{c|cccc} x \backslash y & 1 & 2 & 3 & 4 \\ 1 & 0.08 & 0.08 & 0.12 & 0.12 \\ 2 & 0.03 & 0.12 & 0.06 & 0.09 \\ 3 & 0.06 & 0.02 & 0.02 & 0.10 \\ 4 & 0.01 & 0.01 & 0.07 & 0.01 \end{array}$$

We will consider four decoders:

(a) The naïve decoder. For naïve decoder, $\hat{x}_{\text{naïve}}(y) = y$. Therefore, we can simply copy the values in the y-column into its decoding table.

(b) A DIY decoder defined by $\hat{x}_{\text{DIY}}(y) = 5 - y$. The equation for the decoder is given; so we can simply plug-in each of the possible y values to get $\hat{x}_{\text{DIY}}(y)$.

(c) The MAP decoder.

(d) The ML decoder.

Deriving the MAP and ML decoder follows almost the same recipe: select the max value in each column and read the corresponding x-value. The difference is that the MAP decoder uses the **P** matrix but the ML decoder uses the **Q** matrix.

$$\mathbf{Q} = \begin{array}{c|cccc} x \backslash y & 1 & 2 & 3 & 4 \\ 1 & 0.2 & 0.2 & 0.3 & 0.3 \\ 2 & 0.1 & \mathbf{0.4} & 0.2 & 0.3 \\ 3 & \mathbf{0.3} & 0.1 & 0.1 & \mathbf{0.5} \\ 4 & 0.1 & 0.1 & \mathbf{0.7} & 0.1 \end{array} \qquad \mathbf{P} = \begin{array}{c|cccc} x \backslash y & 1 & 2 & 3 & 4 \\ 1 & \mathbf{0.08} & 0.08 & \mathbf{0.12} & \mathbf{0.12} \\ 2 & 0.03 & \mathbf{0.12} & 0.06 & 0.09 \\ 3 & 0.06 & 0.02 & 0.02 & 0.10 \\ 4 & 0.01 & 0.01 & 0.07 & 0.01 \end{array}$$

Find the decoding table for each of these decoders. Also, find their probabilities of decoding error. Put your answers in the table below.

y	$\hat{x}_{\text{naïve}}(y)$	$\hat{x}_{\text{DIY}}(y)$	$\hat{x}_{\text{MAP}}(y)$	$\hat{x}_{\text{ML}}(y)$
1	1	4	1	3
2	2	3	2	2
3	3	2	1	4
4	4	1	1	3
$P(\mathcal{E})$	0.77	0.79	0.56	0.65

Once a decoder $\hat{x}(y)$ is defined, we can find its corresponding $P(\mathcal{E})$ easily from the **P** matrix:

For each column y in the **P** matrix, circle the element whose corresponding x value is the same as $\hat{x}(y)$;
then, $P(\mathcal{C})$ = the sum of the circled probabilities;
and $P(\mathcal{E}) = 1 - P(\mathcal{C})$.

P =

$\hat{x}_{\text{naive}}(y)$	1	2	3	4
$x \backslash y$	1	2	3	4
1	0.08	0.08	0.12	0.12
2	0.03	0.12	0.06	0.09
3	0.06	0.02	0.02	0.10
4	0.01	0.01	0.07	0.01

$\hat{x}_{\text{DIY}}(y)$	4	3	2	1
$x \backslash y$	1	2	3	4
1	0.08	0.08	0.12	0.12
2	0.03	0.12	0.06	0.09
3	0.06	0.02	0.02	0.10
4	0.01	0.01	0.07	0.01

$\hat{x}_{\text{MAP}}(y)$	1	2	1	1
$x \backslash y$	1	2	3	4
1	0.08	0.08	0.12	0.12
2	0.03	0.12	0.06	0.09
3	0.06	0.02	0.02	0.10
4	0.01	0.01	0.07	0.01

$\hat{x}_{\text{ML}}(y)$	3	2	4	3
$x \backslash y$	1	2	3	4
1	0.08	0.08	0.12	0.12
2	0.03	0.12	0.06	0.09
3	0.06	0.02	0.02	0.10
4	0.01	0.01	0.07	0.01