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ECS 452: Digital Communication Systems **2018/2**

HW 9 — Due: Not Due

Lecturer: Prapun Suksompong, Ph.D.

Problem 1. In a ternary signaling scheme (where signals are expressed with respect to some orthonormal basis), the message S is randomly selected from a constellation $\mathcal{S} = \{-1, 1, 4\}$ with $p_1 = P[S = -1] = 0.41$, $p_2 = P[S = 1] = 0.08$ and $p_3 = P[S = 4] = 0.51$. Find the average signal energy E_s.

Problem 2. Consider a ternary constellation. Assume that the three vectors are equiprobable.

(a) Suppose the three vectors are

$$\mathbf{s}^{(1)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{s}^{(2)} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \text{ and } \mathbf{s}^{(3)} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Find the corresponding average energy per symbol.

(b) Suppose we can shift the above constellation to other location; that is, suppose that the three vectors in the constellation are

$$\mathbf{s}^{(1)} = \begin{pmatrix} 0 - a_1 \\ 0 - a_2 \end{pmatrix}, \mathbf{s}^{(2)} = \begin{pmatrix} 3 - a_1 \\ 0 - a_2 \end{pmatrix}, \text{ and } \mathbf{s}^{(3)} = \begin{pmatrix} 3 - a_1 \\ 3 - a_2 \end{pmatrix}.$$

Find a_1 and a_2 such that corresponding average energy per symbol is minimum.

Problem 3. Let $g(t)$ be band-limited to 100 Hz and $E_g = 8$. A signal set is given in each part below. For rigorous analysis, look at the results from Problem 9 first.

(a) $\{s_m(t) = (2m-5)\, g(t) \cos(2,000\pi t), \quad m = 1,2,3,4\}$
Consider a unit-energy signal $\phi(t) = \alpha g(t) \cos(2,000\pi t)$.

(i) Find the constant α.

(ii) Draw the corresponding constellation (with respect to $\phi(t)$).

(b) $\left\{s_m(t) = g(t) \cos\left(2,000\pi t + \frac{\pi}{2}(m-1)\right), \quad m = 1,2,3,4\right\}$
Consider two orthonormal signals:

$$\phi_1(t) = \alpha_1 g(t) \cos(2\pi f_c t), \quad \text{and} \quad \phi_2(t) = -\alpha_2 g(t) \sin(2\pi f_c t).$$

(i) Find the positive constants α_1 and α_2.

(ii) Draw the corresponding constellation (with respect to $\phi_1(t)$ and $\phi_2(t)$).

(c) $\left\{s_m(t) = 1_{[0,2]}(t)\cos(\pi m t), \quad m = 1, 2, 3\right\}$
Consider three orthonormal signals: $\left\{\phi_m(t) = \alpha_m 1_{[0,2]}(t)\cos(\pi m t), \quad m = 1, 2, 3\right\}$

(i) Find the positive constants α_1, α_2, and α_3.

(ii) Draw the corresponding constellation (with respect to $\phi_1(t)$, $\phi_2(t)$, and $\phi_2(t)$).

Problem 4. In a binary antipodal signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-3, 3\}$ with $p_1 = P[S = -3] = 0.3$ and $p_2 = P[S = 3] = 0.7$. The message is corrupted by an independent additive noise N which is uniform on $[-4, 4]$.

(a) Find the pdf of the noise.

(b) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(c) Evaluate the error probability of the MAP detector.

Problem 5. In a binary antipodal signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-3, 3\}$ with $p_1 = P[S = -3] = 0.3$ and $p_2 = P[S = 3] = 0.7$. The message is corrupted by an independent additive exponential noise N whose pdf is

$$f_N(n) = \begin{cases} \frac{1}{2}e^{-n/2}, & n \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(b) Evaluate the error probability of the MAP detector.

Problem 6. In a ternary signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-1, 1, 4\}$ with $p_1 = P[S = -1] = 0.3 = p_2 = P[S = 1]$ and $p_3 = P[S = 4] = 0.4$. The message is corrupted by an independent additive Gaussian noise $N \sim \mathcal{N}(0, 2)$.

(a) Find the average signal energy[1] E_s.

(b) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

[1]Same as "average symbol energy" or "average energy per symbol" or "average energy per signal"

(c) Indicate the decision regions of the MAP detector in part (b).

(d) Evaluate the error probability of the MAP detector.

Problem 7. In a **standard** quaternary signaling scheme, the message S is equiprobably selected from the alphabet set $\mathcal{S} = \left\{-\frac{3d}{2}, -\frac{d}{2}, \frac{d}{2}, \frac{3d}{2}\right\}$. The message is corrupted by an independent additive exponential noise N whose pdf is

$$f_N(n) = \begin{cases} \lambda e^{-\lambda n}, & n \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the average symbol energy.

(b) Find the average energy per bit.

(c) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(d) Evaluate the error probability of the MAP detector.

(e) Let $\lambda = \frac{1}{\sigma}$. (This is to set $\operatorname{Var} N = \sigma^2$ as in the case for Gaussian noise.) Plot $\frac{E_b}{\sigma^2}$ vs. probability of error $P(\mathcal{E})$. Consider $\frac{E_b}{\sigma^2}$ from -30 to 10 dB.

Extra Questions

Here are some optional questions for those who want more practice.

Problem 8. Suppose $s_1(t) = \text{sinc}(5t)$ and $s_2(t) = \text{sinc}(7t)$. Note that in this class, we define $\text{sinc}(x) = \frac{\sin x}{x}$. Find

(a) E_{s_1},

(b) E_{s_2}, and

(c) $\langle s_1(t), s_2(t) \rangle$.

Hint: Use Parseval's theorem to evaluate the above quantities in the frequency domain. (Review: See 2.43 on p. 22 and Example 2.44 on p. 23 of ECS332 2018 lecture notes and Problem 6 in ECS332 2018 HW3.)

Problem 9. Prove the following facts with the help of Fourier transform.
(Hint: inner product in the frequency domain, Parseval's theorem)

(a) The energy of $p(t) = g(t)\cos(2\pi f_c t + \phi)$ is $E_g/2$.

(b) $g(t)\cos(2\pi f_c t)$ and $-g(t)\sin(2\pi f_c t)$ are orthogonal.

Is there any condition(s) on $g(t)$ that we need to assume?