

ECS 452: Digital Communication Systems

2018/2

HW 8 — Due: May 3, 4 PM

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Instructions

- This assignment has 7 pages.
- (1 pt) Work and write your answers directly on these provided sheets (not on other blank sheet(s) of paper). Hard-copies are distributed in class.
- (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.
- (8 pt) Try to solve all non-optional problems.
- Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Consider a convolutional encoder whose trellis diagram is given in Figure 8.1.

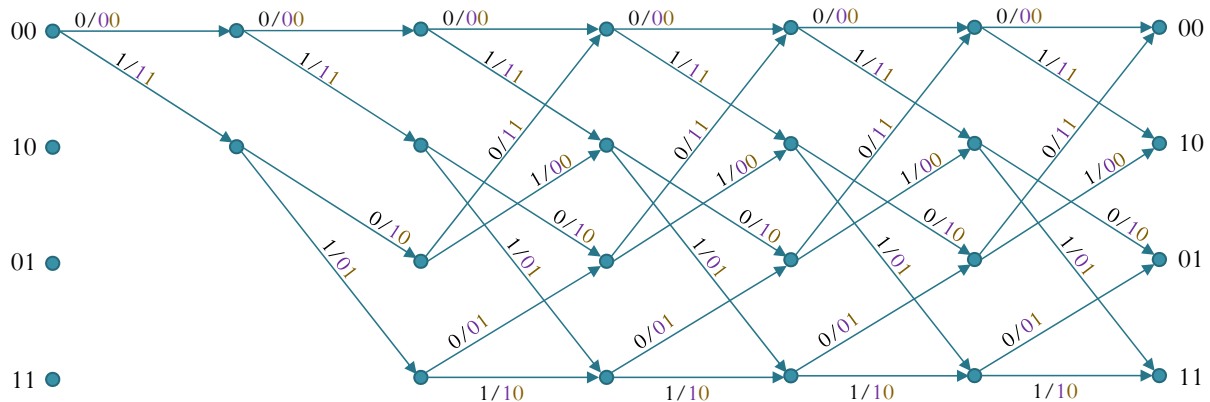


Figure 8.1: State diagram for a convolutional encoder

- Find the code rate

- (b) Suppose the data bits (message) are $\underline{\mathbf{b}} = [0100101]$. Find the corresponding codeword $\underline{\mathbf{x}}$.
- (c) Find the data vector $\underline{\mathbf{b}}$ which gives the codeword $\underline{\mathbf{x}} = [001110111110]$.
- (d) Suppose that we observe $\underline{\mathbf{y}} = [001110000101]$ at the input of the minimum distance decoder. Explain why we can easily find the decoded codeword $\underline{\hat{\mathbf{x}}}$ and the decoded message $\underline{\hat{\mathbf{b}}}$ without applying the Viterbi algorithm.

- (e) Suppose that we observe $\underline{\mathbf{y}} = [010101111110]$ at the input of the minimum distance decoder. Use Viterbi algorithm to find the decoded codeword $\underline{\hat{\mathbf{x}}}$ and the decoded message $\underline{\hat{\mathbf{b}}}$. Show your work on Figure 8.2 below.

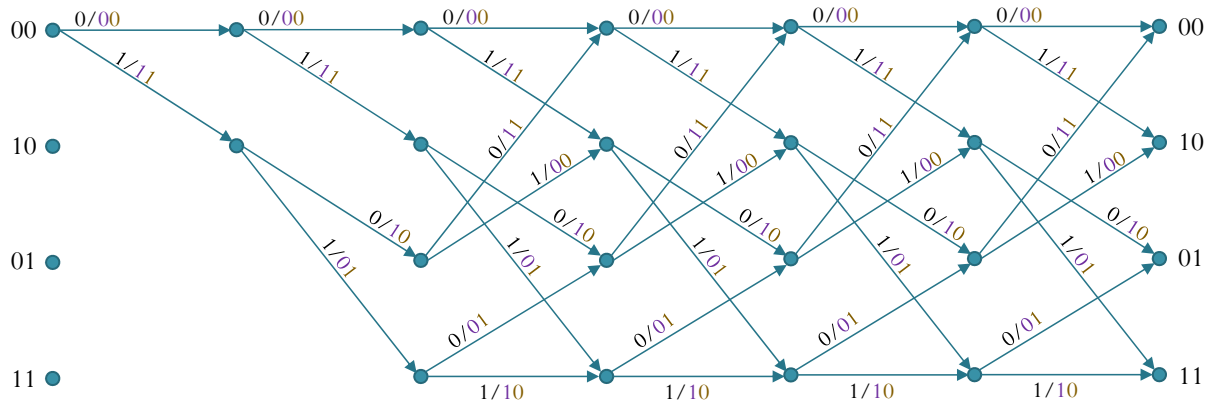


Figure 8.2: State diagram for a convolutional encoder

Make sure that all the running (cumulative) path metric are shown and the discarded branches are indicated at every steps.

Problem 2. Consider four vectors:

$$\mathbf{v}^{(1)} = \begin{pmatrix} +1 \\ -1 \\ +1 \\ 0 \\ -1 \end{pmatrix}, \mathbf{v}^{(2)} = \begin{pmatrix} +1 \\ +1 \\ 0 \\ +1 \\ 0 \end{pmatrix}, \mathbf{v}^{(3)} = \begin{pmatrix} +2 \\ 0 \\ +1 \\ +1 \\ -1 \end{pmatrix}, \text{ and } \mathbf{v}^{(4)} = \begin{pmatrix} +3 \\ +1 \\ +1 \\ +2 \\ -1 \end{pmatrix}.$$

Now, consider two vectors:

$$\mathbf{e}^{(1)} = \frac{1}{2} \begin{pmatrix} +1 \\ -1 \\ +1 \\ 0 \\ -1 \end{pmatrix}, \text{ and } \mathbf{e}^{(2)} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

(a) Show that the two vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are orthonormal.

(b) Find the corresponding vectors $\mathbf{c}^{(1)}$, $\mathbf{c}^{(2)}$, $\mathbf{c}^{(3)}$, and $\mathbf{c}^{(4)}$ that represent $\mathbf{v}^{(1)}$, $\mathbf{v}^{(2)}$, $\mathbf{v}^{(3)}$, and $\mathbf{v}^{(4)}$ in the new coordinate system defined by vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$.

Problem 3. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 8.4. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

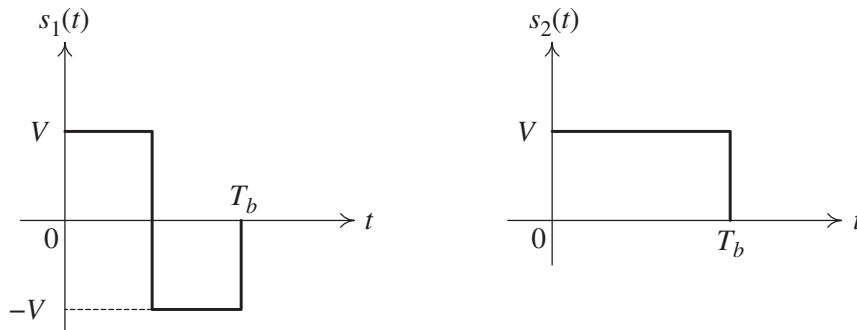


Figure 8.4: Signal set for Problem 3

(a) Find the energy in each signal.

(b) Find the two vectors that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space with respect to the following orthonormal vectors:

$$\phi_1(t) = \begin{cases} \frac{1}{\sqrt{T_b}}, & 0 \leq t \leq \frac{T_b}{2}, \\ -\frac{1}{\sqrt{T_b}}, & \frac{T_b}{2} < t < T_b, \\ 0, & \text{otherwise} \end{cases} \quad \phi_2(t) = \begin{cases} \frac{1}{\sqrt{T_b}}, & 0 \leq t \leq T_b, \\ 0, & \text{otherwise} \end{cases}$$

Also draw the corresponding constellation.

Problem 4. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 8.5. Note that V , α and T_b are some positive constants.

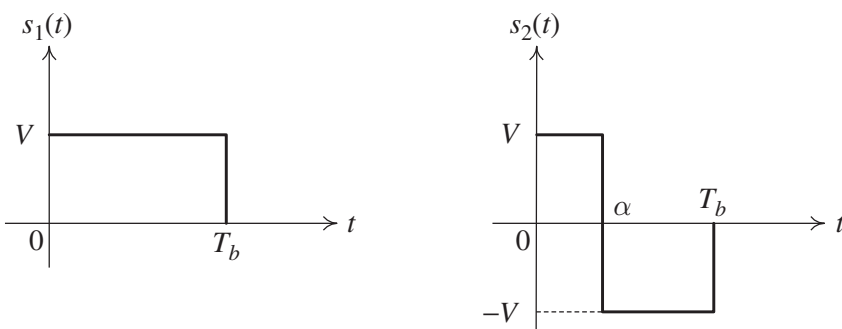


Figure 8.5: Signal set for Problem 4

(a) Find the energy in each signal.

(b) Find $\langle s_1(t), s_2(t) \rangle$.

(c) Consider two orthonormal vectors:

$$\phi_1(t) = \begin{cases} \frac{1}{\sqrt{T_b}}, & 0 < t < T_b, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad \phi_2(t) = \frac{1}{\sqrt{\alpha \left(1 - \frac{\alpha}{T_b}\right)}} \times \begin{cases} 1 - \frac{\alpha}{T_b}, & 0 < t \leq \alpha, \\ -\frac{\alpha}{T_b}, & \alpha < t < T_b, \\ 0, & \text{otherwise} \end{cases}$$

(i) Check that they are orthonormal.

(ii) Find the two vectors $\mathbf{s}^{(1)}$ and $\mathbf{s}^{(2)}$ that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space with respect to the given orthonormal vectors.

(iii) Draw the corresponding constellation when $\alpha = \frac{T_b}{4}$.

- (iv) Draw $\mathbf{s}^{(2)}$ when $\alpha = \frac{k}{10}T_b$ where $k = 1, 2, \dots, 9$.

Extra Question

Here is an optional question for those who want more practice.

Problem 5. Consider a convolutional code generated by the encoder shown in Figure 8.3. Suppose that we observe $\underline{\mathbf{y}} = [110111000110]$ at the input of the minimum distance decoder. Use Viterbi algorithm to find the decoded codeword $\underline{\hat{\mathbf{x}}}$ and the decoded message $\underline{\hat{\mathbf{b}}}$. Caution: The trellis diagram is not the same as the one used in Problem 1.

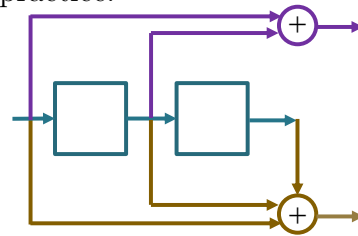


Figure 8.3: Encoder for Problem 6