Name ________________________ ID3 _ _ _

ECS 452: Digital Communication Systems **2018/2**

HW 6 — Due: April 11, 4 PM

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Instructions

(a) This assignment has 6 pages.

(b) (1 pt) Work and write your answers **directly on these provided sheets** (not on other blank sheet(s) of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Consider a single-parity-check linear code. For each of the data block below, find the corresponding codeword.

$\underline{b}$	$\underline{x}$
010	
111	
001	

Problem 2. For each of the codes below, check whether it is a linear code.

(a) $\mathcal{C} = \{000, 001, 100, 101\}$

(b) $\mathcal{C} = \{000, 100, 110, 111\}$

(c) $\mathcal{C} = \{001, 100, 101\}$

Problem 3. Consider a block code whose generator matrix is

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(a) Find the dimension k of this code.

(b) Find its code rate.

(c) Suppose the message is $\underline{\mathbf{b}} = [1\,0\,1]$. Find the corresponding codeword $\underline{\mathbf{x}}$.

(d) For each of the following vectors, indicate whether it is a valid codeword for this code. If yes, find the message $\underline{\mathbf{b}}$ that produces it. If no, state your reason.

(i) $[0\,1\,1\,1\,0\,1]$

(ii) $[1\,1\,0\,1\,1\,1]$

Problem 4. Consider a block code whose codewords are generated by $\underline{\mathbf{x}} = \underline{\mathbf{b}}\mathbf{G}$ where $\underline{\mathbf{b}}$ is the data block and

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Let the row vector $\underline{\mathbf{g}}^{(i)}$ represents the ith row of $\mathbf{G}$. Observe that $\underline{\mathbf{g}}^{(3)} = \underline{\mathbf{g}}^{(1)} \bigoplus \underline{\mathbf{g}}^{(2)}$. Why is this bad?

Problem 5. Consider each of the block codes whose codebooks are provided below. For each code, is the code a linear code that is generated by a generator matrix? If yes, find the corresponding generator matrix. If no, provide a counter-example to support your conclusion.

(a)

$\underline{\mathbf{b}}$			$\underline{\mathbf{c}}$				
0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	0
0	1	0	1	0	1	0	1
0	1	1	1	0	0	1	1
1	0	0	1	1	1	0	0
1	0	1	1	1	0	1	0
1	1	0	0	1	0	0	1
1	1	1	0	1	1	1	1

(b)

$\underline{\mathbf{b}}$			$\underline{\mathbf{c}}$				
0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	0
0	1	0	1	0	1	0	1
0	1	1	1	0	0	1	1
1	0	0	1	1	1	0	0
1	0	1	1	1	1	1	0
1	1	0	0	1	0	0	1
1	1	1	0	1	1	1	1

Problem 6. Consider the following encoding for a systematic linear block code:

- The bit positions that are powers of 2 (1, 2, 4, 8, 16, etc.) are check bits.
- The rest (3, 5, 6, 7, 9, etc.) are filled up with the k data bits.
- Each check bit forces the parity of some collection of bits, including itself, to be even.
  - To see which check bits the data bit in position i contributes to, rewrite i as a sum of powers of 2. A bit is checked by just those check bits occurring in its expansion.

We will consider the case when the codeword's length $n = 7$.

(a) How many bits are check bits?
Hint: How many bit positions are powers of 2?

(b) Find the generator matrix $\mathbf{G}$ for this code.

(c) Find the corresponding parity check matrix $\mathbf{H}$.

Extra Questions

Here are some optional questions for those who want more practice.

Problem 7 (Carlson and Crilly, 2009, P13.2-1)**.** In mathematical analysis, a function $d(\underline{\mathbf{x}}, \underline{\mathbf{y}})$ is a "true" distance if it satisfies all of the following properties:

(i) positivity: $d(\underline{\mathbf{x}}, \underline{\mathbf{y}}) \geq 0$ with equality if and only if $\underline{\mathbf{x}} = \underline{\mathbf{y}}$

(ii) symmetry: $d(\underline{\mathbf{x}}, \underline{\mathbf{y}}) = d(\underline{\mathbf{y}}, \underline{\mathbf{x}})$

(iii) triangle inequality: $d(\underline{\mathbf{x}}, \underline{\mathbf{z}}) \leq d(\underline{\mathbf{x}}, \underline{\mathbf{y}}) + d(\underline{\mathbf{y}}, \underline{\mathbf{z}})$

Is the Hamming distance a "true" distance? (Prove or disprove)

Hint: For the triangle inequality, first consider the number of 1s in $\underline{\mathbf{u}}$, $\underline{\mathbf{v}}$, and $\underline{\mathbf{u}} \bigoplus \underline{\mathbf{v}}$ and confirm that $d(\underline{\mathbf{u}}, \underline{\mathbf{v}}) \leq w(\underline{\mathbf{u}}) + w(\underline{\mathbf{v}})$. Then, from this inequality, replace $\underline{\mathbf{u}}$ by $\underline{\mathbf{x}} \bigoplus \underline{\mathbf{y}}$ and $\underline{\mathbf{v}}$ by $\underline{\mathbf{y}} \bigoplus \underline{\mathbf{z}}$.

Problem 8 (Carlson and Crilly, 2009, P13.2-2 and P13.2-3). Consider a block code. Suppose $\underline{\mathbf{x}}$ is the transmitted codeword and that $\underline{\mathbf{y}}$ is the vector that results when $\underline{\mathbf{x}}$ is received with $i > 0$ bit errors. Use the triangle inequality for the Hamming distance to show that

(a) if the code has $d_{\min} \geq \ell + 1$ and if $i \leq \ell$, then the errors are detectable.

(b) if the code has $d_{\min} \geq 2t + 1$ and if $i \leq t$, then the errors are correctable by the minimum distance decoder.