

**ECS 452: Digital Communication Systems****2018/2****HW 5 — Due: Not Due***Lecturer: Prapun Suksompong, Ph.D.*

**Problem 1.** For each of the DMCs whose corresponding transition probability matrices  $\mathbf{Q}$  are specified below, (i) draw the channel diagram and (ii) compute its capacity  $C$  and the corresponding  $\underline{\mathbf{p}}$  that achieves it.

(a)

$$\mathbf{Q} = \begin{bmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix}$$

(b)

$$\mathbf{Q} = \begin{bmatrix} 0 & 1/5 & 4/5 & 0 & 0 \\ 2/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1. \end{bmatrix}$$

(c)

$$\mathbf{Q} = \begin{bmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{bmatrix}$$

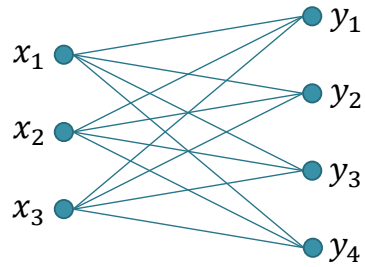


Figure 5.1: Channel diagram for Problem 2.

**Problem 2.** The channel diagram for a DMC is shown in Figure 4.1.

- (a) What are the dimensions of its matrix  $\mathbf{Q}$ ?
  
  
  
  
  
  
  
  
  
  
- (b) Is it possible to find appropriate values for the  $\mathbf{Q}$  matrix of this DMC such that  $C = 1.6$  bpcu? If so, give an example of such  $\mathbf{Q}$ .
  
  
  
  
  
  
  
  
  
  
- (c) Is it possible to find appropriate values for the  $\mathbf{Q}$  matrix of this DMC such that  $C = \log_2 3$  bpcu? If so, give an example of such  $\mathbf{Q}$ .

**Problem 3** (Calculus\*). On many occasions, we have to work with a DMC whose  $\mathbf{Q}$  matrix *does not match* any of our special cases discussed in class. As discussed in class, capacity calculation for these more general cases can be done via treating them as optimization problems in calculus. When  $|\mathcal{X}| = 2$ ,  $\underline{\mathbf{p}} = [p_0 \ 1 - p_0]$ . So, the optimization is over one variable. In general, when  $|\mathcal{X}| = n$ , the optimization is over  $n - 1$  variables.

For each of the DMCs whose transition probability matrices  $\mathbf{Q}$  are specified below, (i) draw the channel diagram and (ii) compute its capacity  $C$  and the corresponding  $\underline{\mathbf{p}}$  that achieves it by first calculating  $I(X; Y)$  for an arbitrary  $\underline{\mathbf{p}}$  (with appropriate dimension) and then set appropriate (partial) derivative(s) to 0 to solve for  $\underline{\mathbf{p}}$ .

(a)

$$\mathbf{Q} = \begin{bmatrix} 1/3 & 2/3 & 0 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

(b)

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

Hint: Let  $\underline{\mathbf{p}} = [p_1, p_2, p_3]$ . Note that the values of  $p_2$  and  $p_3$  only affect  $I(X, Y)$  through  $p_2 + p_3$ , which is  $1 - p_1$ . So, we are back to optimizing over only one variable.

**Problem 4.** The selected examples of the  $\mathbf{Q}$  matrices presented in class are not the only examples in which capacity values can be found without solving the full-blown optimization problems (from which you have suffered in Problem 3). Here is one more example. Consider a DMC whose transition probability matrix  $\mathbf{Q}$  is

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1/3 & 2/3 \end{bmatrix}.$$

(a) Draw the channel diagram.

(b) Compute its capacity  $C$  and the corresponding  $\underline{\mathbf{p}}$  that achieves it.

Hint: Bound the capacity from the dimensions of  $\mathbf{Q}$  then try to find a  $\underline{\mathbf{p}}$  that achieves the bound. Note also that without the last row in the matrix  $\mathbf{Q}$ , the channel is noiseless. One may eliminate a “useless” channel input by not using it at all.

**Problem 5** (Blahut-Arimoto algorithm). A MATLAB function `capacity_blahut` is provided on the course web site. It calculates the capacity  $C = \max_{\underline{\mathbf{p}}} I(\underline{\mathbf{p}}, \mathbf{Q})$  and the corresponding capacity-achieving input pmf  $\underline{\mathbf{p}}$  using Blahut-Arimoto algorithm.

Write a MATLAB script to check your answers in all earlier problems with the help of the provided MATLAB function.