Name ________________________ ID3 _ _ _

ECS 452: Digital Communication Systems **2018/2**

HW 2 — Due: Feb 14, 4 PM

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Instructions

(a) This assignment has 6 pages.

(b) (1 pt) Work and write your answers **directly on these provided sheets** (not on other blank sheet(s) of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. To determine whether a code (or an encoder) is non-singular, uniquely decodable, or prefix-free, it is not necessary to consider the whole codebook. Many questions of this type will give only the collection (or list) of codewords that the code uses.

Consider the code $\{0, 01\}$.

(a) Is it nonsingular?

(b) Is it uniquely decodable?

(c) Is it prefix-free?

Problem 2. In this question, each output string from a DMS is encoded by the following source code:

x	Codeword $c(x)$
'a'	1
'd'	01
'e'	0000
'i'	001
'o'	00010
'u'	00011

(a) Is the code prefix-free?

(b) Is the code uniquely decodable?

(c) Suppose the DMS produces the string 'audio'. Find the output of the source encoder.

(d) Suppose the output of the source encoder is
0 0 0 1 0 0 1 0 0 0 0 0 0 1 0 1 0 0 0 0 1 0 1 0 0 1 0 0 0 0

Find the corresponding source string produced by the DMS. Use "/" to indicate the locations where the sting above is split into codewords.

Problem 3. Consider the random variable X whose support S_X contains seven values:

$$S_X = \{x_1, x_2, \ldots, x_7\}.$$

Their corresponding probabilities are given by

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$p_X(x)$	0.49	0.26	0.12	0.04	0.04	0.03	0.02

(a) Find the entropy $H(X)$.

(b) Find a binary Huffman code for X.

(c) Find the expected codelength for the encoding in part (b).

Problem 4. Find the entropy and the binary Huffman code for the random variable X with pmf

$$p_X(x) = \begin{cases} \frac{x}{21}, & x = 1, 2, \dots, 6, \\ 0, & \text{otherwise.} \end{cases}$$

Also calculate $\mathbb{E}[\ell(X)]$ when Huffman code is used.

Problem 5. Consider a random variable X whose support is $S_X = \{x_1, x_2, \ldots, x_n\}$. Let $p_k = p_X(x_k)$. Then, the entropy of X is

$$H(X) = -\sum_{k=1}^{n} p_X(x_k) \log_2 p_X(x_k) = -\sum_{k=1}^{n} p_k (\log_2 p_k).$$

As discussed in class, observe that the entropy of X does not depend on the specific values $x_1, x_2, \ldots, x_n$ in its support. The entropy is a function of the probability values $p_1, p_2, \ldots, p_n$ in the pmf. Therefore, to calculate entropy, it is enough to specify these probabilities. From this observation, we may write the entropy as a function $H(\underline{\mathbf{p}})$ of a probability vector $\underline{\mathbf{p}}$ constructed from (positive probabilities in) the pmf.

In general, given a probability vector $\underline{\mathbf{p}} = [p_1, p_2, \ldots, p_n]$ whose elements are nonnegative and sum to one, we calculate the corresponding entropy value by

$$H(\underline{\mathbf{p}}) = -\sum_{k=1}^{n} p_k (\log_2 p_k).$$

In each row of the following table, compare the entropy value in the first column with the entropy value in the third column by writing “$>$”, “$=$”, or “$<$” in the second column. Watch out for approximation and round-off error.

$H(\underline{\mathbf{p}})$ when $\underline{\mathbf{p}} = [0.3, 0.7]$.		$H(\underline{\mathbf{q}})$ when $\underline{\mathbf{q}} = [0.8, 0.2]$.
$H(\underline{\mathbf{p}})$ when $\underline{\mathbf{p}} = [0.3, 0.3, 0.4]$.		$H(\underline{\mathbf{q}})$ when $\underline{\mathbf{q}} = [0.4, 0.3, 0.3]$.
$H(X)$ when $p(x) = \begin{cases} 0.3, & x \in \{1, 2\}, \\ 0.2, & x \in \{3, 4\}, \\ 0, & \text{otherwise.} \end{cases}$		$H(\underline{\mathbf{q}})$ when $\underline{\mathbf{q}} = [0.4, 0.3, 0.3]$.

Problem 6. These codes cannot be Huffman codes. Why?

(a) $\{00, 01, 10, 110\}$

(b) $\{01, 10\}$

(c) $\{0, 01\}$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 7. (Optional) The following claim is sometimes found in the literature:

> "It can be shown that the length $\ell(x)$ of the Huffman code of a symbol x with probability $p_X(x)$ is always less than or equal to $\lceil -\log_2 p_X(x) \rceil$".

Even though it is correct in many cases, this claim is not true in general.

Find an example where the length $\ell(x)$ of the Huffman code of a symbol x is greater than $\lceil -\log_2 p_X(x) \rceil$.

Hint: Consider a pmf that has the following four probability values $\{0.01, 0.30, 0.34, 0.35\}$.

Problem 8. (Optional) Construct a random variable X (by specifying its pmf) whose corresponding Huffman code is $\{0, 10, 11\}$.