ECS 452: In-Class Exercise #19

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 26 / 04 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. Consider three vectors: $\vec{\mathbf{v}}^{(1)} = \begin{pmatrix} -1 \\ 5 \\ -5 \\ 1 \end{pmatrix}$, $\vec{\mathbf{v}}^{(2)} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ -2 \end{pmatrix}$, and $\vec{\mathbf{v}}^{(3)} = \begin{pmatrix} 5 \\ -1 \\ 1 \\ -5 \end{pmatrix}$. Let $\vec{\mathbf{e}}^{(1)} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$ and $\vec{\mathbf{e}}^{(2)} = \frac{1}{2}\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$.

a. Show that $\vec{\mathbf{e}}^{(1)}$ and $\vec{\mathbf{e}}^{(2)}$ are orthonormal.

To show that they are orthonormal, we need to check three conditions

① $\|\vec{e}^{(1)}\|^2 = \left(\frac{1}{2}\right)^2\left(1^2+1^2+(-1)^2+(-1)^2\right) = \frac{1}{4}\times 4 = 1 \Rightarrow \|\vec{e}^{(1)}\| = 1$

② $\|\vec{e}^{(2)}\|^2 = \left(\frac{1}{2}\right)^2\left((-1)^2+1^2+(-1)^2+1^2\right) = \frac{1}{4}\times 4 = 1 \Rightarrow \|\vec{e}^{(2)}\| = 1$

both have unit length

③ $\langle \vec{e}^{(1)}, \vec{e}^{(2)} \rangle = \frac{1}{2}\times\frac{1}{2}\times\underbrace{\left(\underbrace{(1)(-1)}_{-1}+\underbrace{(1)(1)}_{1}+\underbrace{(-1)(-1)}_{1}+\underbrace{(-1)(1)}_{-1}\right)}_{0} = 0 \Rightarrow$ orthogonal

ortho-normal

b. Calculate the following inner products:

$\langle \vec{\mathbf{v}}^{(1)}, \vec{\mathbf{e}}^{(1)} \rangle =$

$$\begin{array}{lrrrr} & -1 & 5 & -5 & 1 \\ \times\ \frac{1}{2}(& 1 & 1 & -1 & -1) \\ \hline \frac{1}{2}(& -1 & +5 & +5 & -1) = 4 \end{array}$$

$\langle \vec{\mathbf{v}}^{(1)}, \vec{\mathbf{e}}^{(2)} \rangle =$

$$\begin{array}{lrrrr} & -1 & 5 & -5 & 1 \\ \times\ \frac{1}{2}(& -1 & 1 & -1 & 1) \\ \hline \frac{1}{2}(& 1 & +5 & +5 & +1) = 6 \end{array}$$

$\langle \vec{\mathbf{v}}^{(2)}, \vec{\mathbf{e}}^{(1)} \rangle =$

$$\begin{array}{lrrrr} & 2 & 0 & 0 & -2 \\ \times\ \frac{1}{2}(& 1 & 1 & -1 & -1) \\ \hline \frac{1}{2}(& 2 & +0 & +0 & +2) = 2 \end{array}$$

$\langle \vec{\mathbf{v}}^{(2)}, \vec{\mathbf{e}}^{(2)} \rangle =$

$$\begin{array}{lrrrr} & 2 & 0 & 0 & -2 \\ \times\ \frac{1}{2}(& -1 & 1 & -1 & 1) \\ \hline \frac{1}{2}(& -2 & +0 & -0 & -2) = -2 \end{array}$$

$\langle \vec{\mathbf{v}}^{(3)}, \vec{\mathbf{e}}^{(1)} \rangle =$

$$\begin{array}{lrrrr} & 5 & -1 & 1 & -5 \\ \times\ \frac{1}{2}(& 1 & 1 & -1 & -1) \\ \hline \frac{1}{2}(& 5 & -1 & -1 & +5) = 4 \end{array}$$

$\langle \vec{\mathbf{v}}^{(3)}, \vec{\mathbf{e}}^{(2)} \rangle =$

$$\begin{array}{lrrrr} & 5 & -1 & 1 & -5 \\ \times\ \frac{1}{2}(& -1 & 1 & -1 & 1) \\ \hline \frac{1}{2}(& -5 & -1 & -1 & -5) = -6 \end{array}$$

c. Suppose we use $\vec{\mathbf{e}}^{(1)}$ and $\vec{\mathbf{e}}^{(2)}$ as the new axes. Find the corresponding vectors $\vec{\mathbf{c}}^{(1)}, \vec{\mathbf{c}}^{(2)}$, and $\vec{\mathbf{c}}^{(3)}$ that represent $\vec{\mathbf{v}}^{(1)}, \vec{\mathbf{v}}^{(2)}$, and $\vec{\mathbf{v}}^{(3)}$ in the new coordinate system defined by $\vec{\mathbf{e}}^{(1)}$ and $\vec{\mathbf{e}}^{(2)}$.

$$\vec{c}^{(1)} = \begin{pmatrix} \langle \vec{v}^{(1)}, \vec{e}^{(1)} \rangle \\ \langle \vec{v}^{(1)}, \vec{e}^{(2)} \rangle \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

$$\vec{c}^{(2)} = \begin{pmatrix} \langle \vec{v}^{(2)}, \vec{e}^{(1)} \rangle \\ \langle \vec{v}^{(2)}, \vec{e}^{(2)} \rangle \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\vec{c}^{(3)} = \begin{pmatrix} \langle \vec{v}^{(3)}, \vec{e}^{(1)} \rangle \\ \langle \vec{v}^{(3)}, \vec{e}^{(2)} \rangle \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$