

ECS 452: In-Class Exercise #16

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **05 / 04 / 2019**

Name

ID (last 3 digits)

Prapun

5 5 5

Consider a block code whose generator matrix is

$$\mathbf{G} = \begin{pmatrix} \boxed{1} & \boxed{0} & \boxed{0} & \boxed{1} & \boxed{1} & \boxed{1} \\ \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{1} & \boxed{1} \\ \boxed{0} & \boxed{0} & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{0} \end{pmatrix} \begin{matrix} \mathbf{I}_3 & \mathbf{P} \end{matrix}$$

- a. Find the parity check matrix \mathbf{H} of this code.

$$\mathbf{H} = \begin{pmatrix} \boxed{1} & \boxed{0} & \boxed{1} & \boxed{1} & \boxed{0} & \boxed{0} \\ \boxed{1} & \boxed{1} & \boxed{1} & \boxed{0} & \boxed{1} & \boxed{0} \\ \boxed{1} & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \end{pmatrix} \begin{matrix} \mathbf{P}^T & \mathbf{I}_3 \end{matrix}$$

- b. Suppose we receive $\underline{\mathbf{y}} = 111001$.

- i. Find the syndrome vector $\underline{\mathbf{s}}$

$$\underline{\mathbf{s}} = \underline{\mathbf{y}}\mathbf{H}^T = \begin{pmatrix} \boxed{1} \\ \boxed{1} \\ \boxed{1} \end{pmatrix} + \begin{pmatrix} \boxed{0} \\ \boxed{1} \\ \boxed{1} \end{pmatrix} + \begin{pmatrix} \boxed{1} \\ \boxed{0} \\ \boxed{0} \end{pmatrix} + \begin{pmatrix} \boxed{0} \\ \boxed{1} \\ \boxed{1} \end{pmatrix} = \begin{pmatrix} \boxed{0} \\ \boxed{1} \\ \boxed{1} \end{pmatrix} = (0 \ 1 \ 1)$$

$$\mathbf{H} = \begin{pmatrix} \boxed{1} & \boxed{0} & \boxed{1} & \boxed{1} & \boxed{0} & \boxed{0} \\ \boxed{1} & \boxed{1} & \boxed{1} & \boxed{0} & \boxed{1} & \boxed{0} \\ \boxed{1} & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \end{pmatrix}$$

- ii. Find the decoded codeword $\hat{\underline{\mathbf{x}}}$

The syndrome $\underline{\mathbf{s}}$ is the same as the *second* column of \mathbf{H} .

Therefore, $\hat{\underline{\mathbf{e}}} = (0 \ 1 \ 0 \ 0 \ 0 \ 0)$ and

$$\hat{\underline{\mathbf{x}}} = \underline{\mathbf{y}} - \hat{\underline{\mathbf{e}}} = \underline{\mathbf{y}} \oplus \hat{\underline{\mathbf{e}}} = (1 \ 0 \ 1 \ 0 \ 0 \ 1).$$

- iii. Find the decoded message $\hat{\underline{\mathbf{b}}}$.

From \mathbf{G} , we have \mathbf{I}_3 in the front; so, given a codeword $\underline{\mathbf{x}}$, the message $\underline{\mathbf{b}}$ corresponding to this codeword is given by its first three bits. Here, the decoded codeword is

$\hat{\underline{\mathbf{x}}} = (1 \ 0 \ 1 \ 0 \ 0 \ 1)$. Therefore, the corresponding decoded message is $\hat{\underline{\mathbf{b}}} = (1 \ 0 \ 1)$.