ECS 452: In-Class Exercise # 15

Date: **04** / **04** / 2019	
Name	ID (last 3 digits)
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Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Consider a linear block code that uses *parity checking on a square array*:

First, we use the provided definition to write down the equations that produce the parity bits. This definition is exactly the same as the one given in lecture when we defined parity checking on a square array

b_1	b_2	p_1
b_3	b_4	p_2
p_3	p_4	

$p_1 = b_1 \oplus b_2$

$p_2 = b_3 \oplus b_4$

$p_3 = b_1 \oplus b_3$

$p_4 = b_2 \oplus b_4$

2.

Each parity bit p_i is calculated such that the corresponding row or column has even parity.

Suppose the following bits arrangement is used in the codeword:

$$\underline{\mathbf{x}} = (b_1 \quad p_1 \quad p_2 \quad b_2 \quad b_3 \quad p_3 \quad b_4 \quad p_4).$$

$p_1 = b_1 \oplus b_2$, $p_2 = b_3 \oplus b_4$, $p_3 = b_1 \oplus b_3$, $p_4 = b_2 \oplus b_4$

a. Find the generator matrix $\mathbf{G}$.

Recall that the 1s and 0s in the j^{th} column of **G** tells which positions of the data bits are combined ($\oplus$) to produce the j^{th} bit in the codeword.

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

b. Find the codeword for the message $\underline{\mathbf{b}} = [1 \quad 0 \quad 1 \quad 0]$.

Method 1: First, we fill out the array above with the message. Then, we calculate the parity bits.

1	0	p_1
1	0	p_2
p_3	p_4	

→

1	0	1
1	0	1
0	0	

The codeword can be read directly from the array: $\underline{\mathbf{x}} = (1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0)$.

Method 2: It is still true that $\underline{\mathbf{x}} = \underline{\mathbf{b}}\mathbf{G}$. Therefore, we can still use our old technique: to find $\underline{\mathbf{x}}$ when $\underline{\mathbf{b}} = [1 \quad 0 \quad 1 \quad 0]$, we simply need to add the first and the third rows of $\mathbf{G}$. This also gives $\underline{\mathbf{x}} = (1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0)$.

c. Find the parity-check matrix $\mathbf{H}$.

We look at two parts of $\mathbf{G}$: the message part and the parity part.

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

The parity part from $\mathbf{G}$ is transposed and put into the message positions (columns). The remaining columns are filled in by an identity matrix.

$$\mathbf{H} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$