

ECS 452: In-Class Exercise # 15

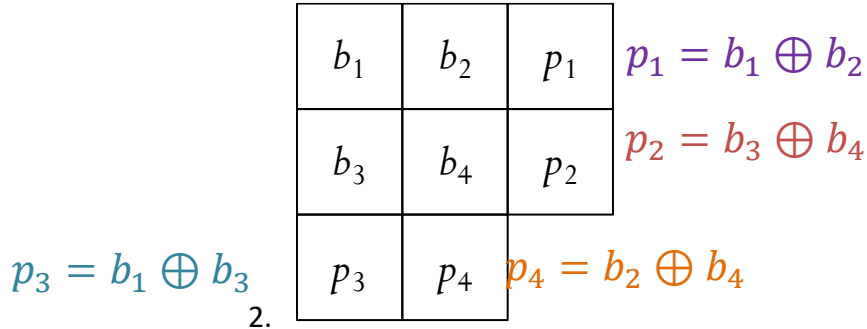
Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 04 / 04 / 2019			
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1. Consider a linear block code that uses *parity checking on a square array*:

First, we use the **provided definition** to write down the equations that produce the parity bits. This definition is exactly the same as the one given in lecture when we defined parity checking on a square array



Each parity bit p_i is calculated such that the corresponding row or column has even parity.

Suppose the following bits arrangement is used in the codeword:

$$\underline{x} = (b_1 \quad p_1 \quad p_2 \quad b_2 \quad b_3 \quad p_3 \quad b_4 \quad p_4).$$

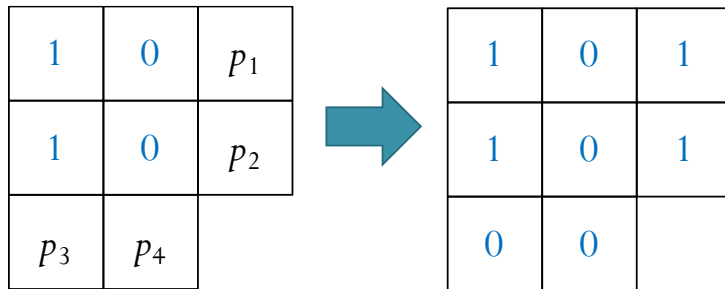
a. Find the generator matrix \mathbf{G} .

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Recall that the 1s and 0s in the j^{th} column of \mathbf{G} tells which positions of the data bits are combined (\oplus) to produce the j^{th} bit in the codeword.

b. Find the codeword for the message $\underline{b} = [1 \ 0 \ 1 \ 0]$.

Method 1: First, we fill out the array above with the message. Then, we calculate the parity bits.



The codeword can be read directly from the array: $\underline{x} = (1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0)$.

Method 2: It is still true that $\underline{x} = \underline{b}\mathbf{G}$. Therefore, we can still use our old technique: to find \underline{x} when

$\underline{b} = [1 \ 0 \ 1 \ 0]$, we simply need to add the first and the third rows of \mathbf{G} . This also gives $\underline{x} = (1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0)$.

c. Find the parity-check matrix \mathbf{H} .

We look at two parts of \mathbf{G} : the message part and the parity part.

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

The parity part from \mathbf{G} is transposed and put into the message positions (columns). The remaining columns are filled in by an identity matrix.

$$\mathbf{H} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$