ECS 452: In-Class Exercise # 13

Date: 22 / 03 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. For each code given below, check whether the code is linear.

$\underline{x}^{(1)}$ $\underline{x}^{(2)}$ $\mathcal{C}$ $\underline{x}^{(3)}$ $\underline{x}^{(4)}$	Linear?
{10000, 01100, 10111, 01011}	No
{10001, 11010, 01111, 00100}	No
{10100, 01001, 10011, 01110}	No
{00110, 01101, 11011, 10000}	No
{00000, 01110, 10011, 11101}	Yes

These codes do not contain the all-zero vector. Therefore, they are not linear.

For the last code, one can check that $\underline{x}^{(i)} \oplus \underline{x}^{(j)} \in \mathcal{C}$ for any i, j.

Alternatively, we have ① $\underline{0} \in \mathcal{C}$

② $\underline{x}^{(2)} \oplus \underline{x}^{(3)} = \underline{x}^{(4)}$ $\Rightarrow$ Note that this implies $\left\{ \begin{array}{l} \underline{x}^{(2)} \oplus \underline{x}^{(4)} = \underline{x}^{(3)} \\ \underline{x}^{(3)} \oplus \underline{x}^{(4)} = \underline{x}^{(2)} \end{array} \right\}$ as well.

2. A linear block code uses the following generator matrix $\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

   a. Find the codeword length n = #columns = 4

   b. Find the codeword for the message $\underline{\mathbf{b}} = [1 \quad 0]$

   $[1\ 0]\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = [1\ 1\ 0\ 1]$

   Direct multiplication like this is OK when we only have to find one codeword. However, in the next part, we need to find all of them; so we will use other methods.

   c. Find the codebook for this code.

   Method 1 : $\underline{x} = \sum_i b_i \underline{g}^{(i)}$ where b_i = the i^{th} element in $\underline{b}$ and $\underline{g}^{(i)}$ = the i^{th} row of $\mathbf{G}$

$\underline{\mathbf{b}}$ ($b_1 b_2$)	$\underline{\mathbf{x}}$
00	0000 $\leftarrow 0 \cdot \underline{g}^{(1)} \oplus 0 \cdot \underline{g}^{(2)} = \underline{0}$
01	1010 $\leftarrow 0 \cdot \underline{g}^{(1)} \oplus 1 \cdot \underline{g}^{(2)} = \underline{g}^{(2)}$
10	1101 $\leftarrow 1 \cdot \underline{g}^{(1)} \oplus 0 \cdot \underline{g}^{(2)} = \underline{g}^{(1)}$
11	0111 $\leftarrow 1 \cdot \underline{g}^{(1)} \oplus 1 \cdot \underline{g}^{(2)} = \underline{g}^{(1)} \oplus \underline{g}^{(2)}$

   Method 2 : $\underline{b} = [b_1 \ \ b_2]$

   $\underline{x} = \underline{b}\mathbf{G}$

   $= [\underbrace{b_1 \oplus b_2}\ \ b_1\ \ b_2\ \ b_1]$

   The first bit is the sum of b_1 and b_2.

   The 2nd and 4th bits are simply b_1.

   The 3rd bit is b_2.