

ECS 452: In-Class Exercise # 12

Instructions

1. Separate into groups of no more than three persons
2. Do not panic.

Date: 22 / 03 / 2019			
Name			ID (last 3 digits)
Prapun			5 5 5

1. Assume GF(2). Calculate the following quantities:

a. $1 \oplus 0 = 1$

b. $1 \oplus 1 \oplus 1 = (1 \oplus 1) \oplus 1 = 0 \oplus 1 = 1$

c. $1 \cdot 0 = 0$

d. $1 \cdot 0 \cdot 1 = (1 \cdot 0) \cdot 1 = 0 \cdot 1 = 0$

e. $[0 \ 1 \ 0] \oplus [1 \ 1 \ 0] = [0 \oplus 1 \ 1 \oplus 1 \ 0 \oplus 0] = [1 \ 0 \ 0]$

f. $[1 \ 1 \ 0] \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = [(1 \cdot 1) \oplus (1 \cdot 1) \oplus (0 \cdot 0) \quad (1 \cdot 0) \oplus (1 \cdot 1) \oplus (0 \cdot 1)] = [1 \oplus 1 \oplus 0 \quad 0 \oplus 1 \oplus 0] = [0 \ 1]$

Alternatively, multiplying by $[1 \ 1 \ 0]$ means we simply add the first two rows of $\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$.

2. Fill in the blanks:

$[_ _ _] \oplus [1 \ 1 \ 0] = [0 \ 0 \ 0]$

$[_ _ _] = [1 \ 1 \ 0]$

$$\begin{aligned} \underline{a} \oplus \underline{b} &= \underline{0} \\ \underline{a} \oplus \underline{b} \oplus \underline{b} &= \underline{0} \oplus \underline{b} \\ &\underline{0} \\ \underline{a} &= \underline{b} \end{aligned}$$

If we define the “negative” of \underline{b} to be the vector that give $\underline{0}$ when added to \underline{b} . Here, in GF(2), we see that “negative” of \underline{b} is \underline{b} itself.

3. A codeword $[1 \ 1 \ 1 \ 0]$ is sent over the BSC. Suppose the error pattern is $\underline{e} = [0 \ 1 \ 1 \ 1]$.

Find the observed vector at the receiver.

$\underline{y} = \underline{x} \oplus \underline{e} = [1110] \oplus [0111] = [1001]$

Alternatively, from the error pattern, we know that the last three bits of \underline{x} will be flipped by the BSC. So, \underline{y} is constructed accordingly.

4. A codeword $[1 \ 1 \ 1 \ 0]$ is sent over the BSC. Suppose the observed vector at the receiver is

$\underline{y} = [0 \ 0 \ 1 \ 1]$. Find the error pattern.

From $\underline{y} = \underline{x} \oplus \underline{e}$, we have $\underline{e} = \underline{x} \oplus \underline{y} = [1110] \oplus [0011] = [1101]$

$$\begin{aligned} \underline{y} &= \underline{x} \oplus \underline{e} \\ \underline{x} \oplus \underline{y} &= \underline{x} \oplus \underline{x} \oplus \underline{e} \\ &\underline{0} \\ \underline{x} \oplus \underline{y} &= \underline{e} \end{aligned}$$

Alternatively, recall that the error pattern indicates the locations of error. Here, all bits of \underline{x} and \underline{y} are different except the third bit. So, the error pattern should be $[1101]$.

5. A codeword is sent over the BSC.

Suppose the observed vector at the receiver is $\underline{y} = [0 \ 0 \ 1 \ 1]$ and the error pattern is $\underline{e} = [0 \ 1 \ 1 \ 1]$.

Find the transmitted codeword.

From $\underline{y} = \underline{x} \oplus \underline{e}$, we have $\underline{x} = \underline{y} \oplus \underline{e} = [0011] \oplus [0111] = [0100]$

$$\begin{aligned} \underline{y} &= \underline{x} \oplus \underline{e} \\ \underline{y} \oplus \underline{e} &= \underline{x} \oplus \underline{e} \oplus \underline{e} \\ &\underline{0} \\ \underline{y} \oplus \underline{e} &= \underline{x} \end{aligned}$$

Alternatively, the error pattern says that all bits are received incorrectly except the first bit. Therefore, to recover \underline{x} , we need to flip all bits of \underline{y} except the first bit.