ECS 452: In-Class Exercise # 7

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **21** / **02** / 2019			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. Consider a DMC whose transition matrix $\mathbf{Q}$ and joint pmf matrix $\mathbf{P}$ are given below.

$$\mathbf{Q} = \begin{array}{c|cccc} x \backslash y & 1 & 2 & 3 & 4 \\ 1 & 0.3 & 0.4 & 0.2 & 0.1 \\ 2 & 0.2 & 0.5 & 0.1 & 0.2 \\ 3 & 0.1 & 0.3 & 0.3 & 0.3 \end{array} \qquad \mathbf{P} = \begin{array}{c|cccc} x \backslash y & 1 & 2 & 3 & 4 \\ 1 & 0.12 & 0.16 & 0.08 & 0.04 \\ 2 & 0.10 & 0.25 & 0.05 & 0.10 \\ 3 & 0.01 & 0.03 & 0.03 & 0.03 \end{array}$$

a) Find the MAP detector. Put your answer in the decoding table below.
Also find the corresponding error probability.

y	$\hat{x}_{\text{MAP}}(y)$
1	1
2	2
3	1
4	2

$P(C) = 0.12+0.25+0.08+0.10 = 0.55$

$P(\varepsilon) = 1-P(C) = 1-0.55 = 0.45$

b) Find the ML detector. Put your answer in the decoding table below.
Also find the corresponding error probability.

y	$\hat{x}_{\text{ML}}(y)$
1	1
2	2
3	3
4	3

$P(C) = 0.12+0.25+0.03+0.03 = 0.43$

$P(\varepsilon) = 1-P(C) = 1-0.43 = 0.57$

c) Find the pmf $p(x)$ of the channel input X.

Recall that to get the P matrix from the Q matrix, we multiply each row of the Q matrix by the corresponding p(x). So, to get p(x), we simply divides each row of the P matrix by the corresponding row in the Q matrix. (In fact, only one representative from each row is enough.)

first column of the P matrix

$0.12/0.3 = 0.4$
$0.10/0.2 = 0.5$
$0.01/0.1 = 0.1$

first column of the Q matrix

$$p(x) = \begin{cases} 0.4, & x = 1, \\ 0.5, & x = 2, \\ 0.1, & x = 3, \\ 0, & \text{otherwise} \end{cases}$$

Alternatively, recall, from ECS 315, that once the P matrix is there, one can get the pmf of X by summing along each row of the P matrix.