ECS 452: In-Class Exercise # 6

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **15** / **02** / 2019			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. Consider a DMC whose transition matrix $\mathbf{Q}$ is

$$\mathbf{Q} = \begin{array}{c} x\backslash y \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.3 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.3 \end{bmatrix} \end{array} \xrightarrow[\times 0.5]{\times 0.4,\ \times 0.1} \begin{array}{c} x\backslash y \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.12 & 0.16 & 0.08 & 0.04 \\ 0.10 & 0.25 & 0.05 & 0.10 \\ 0.01 & 0.03 & 0.03 & 0.03 \end{bmatrix} \end{array} = \mathbf{P}$$

Σ Σ Σ Σ

$[0.23 \quad 0.44 \quad 0.16 \quad 0.17]$

Suppose the input probability vector is $\underline{\mathbf{p}} = [0.4 \quad 0.5 \quad 0.1]$.

a) Find the joint pmf matrix $\mathbf{P}$. Put your answer next to the $\mathbf{Q}$ matrix above.

b) Find the output probability vector $\underline{\mathbf{q}}$.

$[0.23 \quad 0.44 \quad 0.16 \quad 0.17]$

c) Suppose the naïve decoder is used. Find the corresponding $P(\mathcal{E})$.

$\hat{x} = y$

$$\mathbf{P} = \begin{array}{c} x\backslash y \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.12 & 0.16 & 0.08 & 0.04 \\ 0.10 & 0.25 & 0.05 & 0.10 \\ 0.01 & 0.03 & 0.03 & 0.03 \end{bmatrix} \end{array}$$

$P(\mathcal{C}) = 0.12 + 0.25 + 0.03$

$= 0.40$

$P(\mathcal{E}) = 1 - P(\mathcal{C}) = 1 - 0.40 = 0.60$

d) Suppose the following decoder is used. Find the corresponding $P(\mathcal{E})$.

y	$\hat{x}(y)$
1	3
2	1
3	1
4	3

$\hat{x}$: 3 1 1 3

$$\mathbf{P} = \begin{array}{c} x\backslash y \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.12 & 0.16 & 0.08 & 0.04 \\ 0.10 & 0.25 & 0.05 & 0.10 \\ 0.01 & 0.03 & 0.03 & 0.03 \end{bmatrix} \end{array}$$

$P(\mathcal{C}) = 0.01 + 0.16 + 0.08 + 0.03$

$= 0.28$

$P(\mathcal{E}) = 1 - P(\mathcal{C}) = 1 - 0.28$

$= 0.72$

e) Suppose the decoder is $\hat{x}(y) = 2.5 - |y - 2.5|$

Find the corresponding $P(\mathcal{E})$.

y	$y-2.5$	$2.5-\lvert y-2.5\rvert$
1	-1.5	1
2	-0.5	2
3	0.5	2
4	1.5	1

$$\mathbf{P} = \begin{array}{c} x\backslash y \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.12 & 0.16 & 0.08 & 0.04 \\ 0.10 & 0.25 & 0.05 & 0.10 \\ 0.01 & 0.03 & 0.03 & 0.03 \end{bmatrix} \end{array}$$

$P(\mathcal{C}) = 0.12 + 0.25 + 0.05 + 0.04$

$= 0.46$

$P(\mathcal{E}) = 1 - 0.46 = 0.54$