ECS 452: In-Class Exercise # 5

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **14** / **02** / 2019			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. No need to provide any explanation for this question.
   Consider a DMC whose samples of input and output are provided below

```
x: 1 1 1 0 1 0 1 0 1 1 0 1 1 0 0
y: 1 1 1 0 1 0 1 1 0 1 1 0 1 1 0
```

Estimate the following quantities:

a. $\mathcal{X} = \{0, 1\}$ (channel input alphabet = support of X)

b. $P[X = 0] = \frac{6}{15} = \frac{2}{5} = 0.4$

c. $p(1) \equiv P[X = 1] = 1 - P[X = 0] = 1 - \frac{2}{5} = \frac{3}{5} = 0.6$

d. $p_Y(0) \equiv P[Y = 0] = \frac{5}{15} = \frac{1}{3}$

e. $\underline{p} \equiv [p(0)\ p(1)] = [\frac{2}{5}\ \frac{3}{5}] = [0.4\ 0.6]$

f. $q(1) = P[Y = 1] = 1 - P[Y = 0] = 1 - \frac{1}{3} = \frac{2}{3}$

g. $P[Y = 0 | X = 0] = \frac{3}{6} = \frac{1}{2} = 0.5$

h. $p_{Y|X}(1|0) \equiv P[Y = 1 | X = 0]$
$= 1 - P[Y = 0 | X = 0] = 1 - 0.5 = 0.5$

i. $Q(0|1) \equiv P[Y = 0 | X = 1] = \frac{2}{9}$

j. $Q(1|1) \equiv P[Y = 1 | X = 1]$
$= 1 - P[Y = 0 | X = 1] = 1 - \frac{2}{9} = \frac{7}{9}$

k. Matrix **Q**

x \ y	0	1
0	0.5	0.5
1	2/9	7/9

l. $P[X = 0, Y = 0] = \frac{3}{15} = \frac{1}{5} = 0.2$

Alternative method:

$P[\underbrace{X = 0}_{A}, \underbrace{Y = 0}_{B}] = P(A \cap B) = P(A)P(B|A)$

$= P[X = 0]\, P[Y = 0 | X = 0]$

$= \underbrace{0.4}_{\text{from part (b)}} \times \underbrace{0.5}_{\text{from part (g)}} = 0.2$