ECS 452: In-Class Exercise # 4

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **08** / **02** / 2019			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. Write each of the following quantities in the form X.XXX (possibly with the help of your calculator).
   a. $-\log_2(1/128) = -\log_2\left(\frac{1}{2^7}\right) = -\log_2(2^{-7}) = -(-7)\log_2 2 = 7.000$

   b. $-\log_2(0.6) \approx 0.737$

   Method 1

   $-\log_2 a = \frac{-\log_e a}{\log_e 2} = -\frac{\ln(0.6)}{\ln(2)} \approx -\frac{-0.5108}{0.6931} \approx 0.7370$

   Method 2

   $-\log_2 a = \frac{-\log_{10} a}{\log_{10} 2} = -\frac{\log_{10}(0.6)}{\log_{10}(2)} \approx -\frac{-0.2218}{0.3010} \approx 0.7369$

   c. $-(0.4)\log_2(0.4) - (0.6)\log_2(0.6) \approx 0.971$

   ($\log_2(0.4) \approx -1.3219$, $-(0.4)\log_2(0.4) \approx 0.5288$; $\log_2(0.6) \approx -0.7370$, $-(0.6)\log_2(0.6) \approx 0.4422$)

2. In each part below, we consider a random variable X which has five possible values. The probability for each possible value is listed in the provided table. Calculate the corresponding entropy value.
   a.

x	E	L	M	N	O
$p(x)$	0.25	0.25	0.25	0.125	0.125

$$H(X) = -\sum_x p(x)\log_2 p(x) = -3\times\frac{1}{4}\log_2\frac{1}{4} - 2\times\frac{1}{8}\log_2\frac{1}{8}$$
$$= -3\times\frac{1}{4}\times(-2) - 2\times\frac{1}{4}\times(-3) = \frac{9}{4} = 2.25 \text{ [bits]}$$

   b.

x	E	L	M	N	O
$p(x)$	0.1	0.1	0.2	0.2	0.4

$$H(x) = -2\times0.1\times\log_2 0.1 - 2\times0.2\log_2 0.2 - 0.4\log_2 0.4 \approx 2.1219 \text{ bits}$$
($\log_2 0.1 \approx -3.3219$, $\log_2 0.2 \approx -2.3219$, $\log_2 0.4 \approx -1.3219$)

   c.

x	E	L	M	N	O
$p(x)$	0.42	0.17	0.08	0.08	0.25

$$H(X) = -0.42\log_2 0.42 - 0.17\log_2 0.17 - 2\times0.08\log_2 0.08 - 0.25\log_2 0.25 \approx 2.0433 \text{ bits}$$
($\log_2 0.42 \approx -1.2515$, $\log_2 0.17 \approx -2.5564$, $\log_2 0.08 \approx -3.6439$, $\log_2 0.25 = -2$)