## ECS 452: In-Class Exercise \# 3

## Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups. Only one submission is needed for each group.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

| Date: $\mathbf{0 8} / \mathbf{0 2 / 2 0 1 9}$ |  |  |  |
| :--- | :--- | :--- | :--- |
| Name | ID |  |  |
| Prapun | 5 | 5 | 5 |
|  |  |  |  |
|  |  |  |  |

1. A discrete memoryless source emits three possible messages Yes, No, and OK with probabilities 0.2 and 0.3 , and 0.5 , respectively.
a. Find the expected codeword length when Huffman binary code is used without extension.

The grouping orders are indicated by circled numbers.


$$
0.2 \times 2+0.3 \times 2+0.5 \times 1=1.5 \text { bits per source symbol }
$$

Remark: The problem does not ask us to find the codewords. Only the codeword lengths are needed. Once the tree is formed, we can read the codeword lengths directly.
b. Find the codeword lengths when Huffman binary code with second-order extension is used to encode this source. Put the values of the corresponding probabilities and the codeword lengths in the table below. (Note that, for brevity, we use Y,N,K to represent Yes, No, and OK, respectively.)

| $x_{1} x_{2}$ | $p_{X_{1}, X_{2}}\left(x_{1}, x_{2}\right)$ |  | $\ell\left(x_{1}, x_{2}\right)$ |
| :---: | :---: | :---: | :---: |
| YY | $0.2 \times 0.2=0.04$ | The grouping orders are indicated by circled numbers. | 4 |
| YN | $0.2 \times 0.3=0.06$ | Note that there are many possible solution. This is just | 4 |
| YK | $0.2 \times 0.5=0.10$ | one of them. <br> Remark: The problem does | 3 |
| NY | $0.3 \times 0.2=$ | not ask us to find the codewords. Only the codeworc | 4 |
| NN | $0.3 \times 0.3=0.09$ | lengths are needed. Once the tree is formed, we can read the | 4 |
| NK | $0.3 \times 0.5=0.15$ | codeword lengths directly. | 3 |
| KY | $0.5 \times 0.2=0.10$ |  | 3 |
| KN | $0.5 \times 0.3=0.15$ |  | 3 |
| KK | $0.5 \times 0.5=0.25$ |  | 2 |

Note that even when the Huffman'recipe is followed strictly, there are many possible
c. Find $L_{2}$. solutions. For example, at Step 3, there are three choices of 0.1 that we can choose.
(This is the expected codeword length per source symbol of the Huffman binary code for the second-order extension of this source.)

$$
\begin{aligned}
& (0.04+0.06+0.06+0.09) \times 4+(0.10+0.15+0.10+0.15) \times 3+0.25 \times 2 \\
& =0.25 \times 4+0.5 \times 3+0.5=3 \text { bits per two source symbols. }
\end{aligned}
$$

$L_{2}=\frac{3}{2}=1.5$ bits per source symbol

