ECS 452: In-Class Exercise # 2

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **25** / **01** / 2019			
Name	ID (last 3 digits)		
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1. Consider a DMS whose source alphabet is {E,L,M,N,O}.
   The probabilities for these five symbols are shown in the table below:

x	E	L	M	N	O
$p(x)$	0.1	0.1	0.2	0.2	0.4

Consider two codes (for source coding) below.
The left column is for Code A. The right column is for Code B.
The first row defines these codes via their codebooks.

Codebook for Code A

x	E	L	M	N	O
$c(x)$	101	110	111	011	100

Is Code A prefix-free?
Yes, no codeword is a prefix of another codeword.
Observation: Any fixed-length non-singular codes are also prefix-free.

Suppose the DMS above is encoded by Code A.
Find the expected codeword length.

The length of all code word is 3.
Therefore,

$\mathbb{E}[\ell(X)] =$ 3 bits per source symbol

Codebook for Code B

x	E	L	M	N	O
$c(x)$	0	100	1010	1011	11
$\ell(x)$	1	3	4	4	2

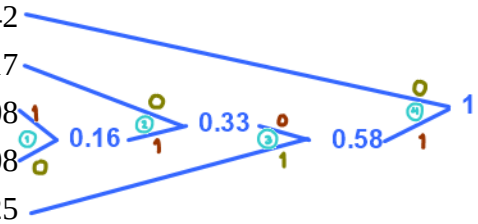
Is Code B prefix-free?
Yes, no codeword is a prefix of another codeword.
Remark: Some codewords have other codewords as their suffixes. However, we only consider prefix, not suffix.

Suppose the DMS above is encoded by Code B.
Find the expected codeword length.

$\mathbb{E}[\ell(X)] = 0.1(1+3) + 0.2(4+4) + 0.4 \times 2$
$= 0.4 + 1.6 + 0.8$
$= 2.8$ bits per source symbol

2. Consider a random variable X which has five possible values. Their probabilities are shown in the table below.

x	$p_X(x)$	$c(x)$	$\ell(x)$
E	0.42	0	1
L	0.17	100	3
M	0.08	1011	4
N	0.08	1010	4
O	0.25	11	2



The tree can be contruct by following Huffman's recipe.
The grouping orders are indicated by circled numbers.
The code symbols on each branch are forced by having to make 1011 the codeword for M.

   a. Find a binary Huffman code (without extension) for this random variable.
      Put the values of the codewords and the codeword lengths in the table above.
      Note that the codeword for the source symbol "M" is required to be 1011.
   b. Find the expected codeword length when Huffman coding is used (without extension).

$= 0.42x1 + 0.17x3 + (0.08+0.08)x4 + 2x0.25$
$= 0.42 + 0.51 + 0.64 + 0.50$
$= 2.07$ [bits per source symbol]