ECS 452: In-Class Exercise # 10

Date: 08/ 03/ 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. For each of the following DMC's probability transition matrices **Q**, (i) indicate whether the corresponding DMC is weakly symmetric (Yes or No), (ii) evaluate the corresponding capacity value (your answer should be of the form X.XXXX), and (iii) specify the channel input pmf (a row vector **p**) that achieves the capacity.

Check that
(1) all the rows of Q are permutations of each other
and
(2) all the column sums are equal

$-0.4\log_2 0.4 - 0.6\log_2 0.6$
≈ 0.9710
This is computed in the previous exercise already.

crossover probability p **Q**	Weakly Symmetric?	C	**p**
$\begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$ This is the Q matrix for a BSC.	Yes. BSC is symmetric and hence weakly symmetric.	For BSC, $C = 1 - h(p)$ (binary entropy function) $= 1 - h(0.4)$ $\approx 1 - 0.9710$ ≈ 0.0290 [bpcu]	C is achieved by uniform X on $\mathcal{X}$. $\underline{p} = \left[\frac{1}{2}\ \frac{1}{2}\right]$
r: $\begin{bmatrix} 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 \end{bmatrix}$	(1) ✓ (2) ✓ Yes	$\log_2\lvert\mathcal{Y}\rvert - H(\underline{r})$ $= \log_2 4 - H([0.5\ 0.5])$ $= 2 - 1 = 1$ [bpcu]	C is achieved by uniform X on $\mathcal{X}$. $\underline{p} = \left[\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\right]$
$\begin{bmatrix} 0.5 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0.5 & 0 & 0.5 & 0 \end{bmatrix}$	(1) × (2) × No	Note that there is only one non-zero element in each column ⇒ This is NO^2 channel ⇒ $C = \log_2\lvert\mathcal{X}\rvert$ $= \log_2 4$ ≈ 2 [bpcu]	⇒ C is achieved by uniform X. $\underline{p} = \left[\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\right]$
$\begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$	(1) ✓ (2) × No	Note that all the rows of Q are the same ⇒ $Q(y\|x)$ does not depend on x ⇒ $X \perp\!\!\!\perp Y$ ⇒ $I(X;Y) = 0$ for any $p(x)$ ⇒ $C = 0.0000$ [bpcu]	Any $\underline{p}$ will give the same $I(X;Y) = C = 0$.

Specifically, any $\underline{p} = [p_1\ p_2]$ such that $p_1, p_2 \geq 0$ and $p_1 + p_2 = 1$ will work.