ECS 452: In-Class Exercise # 11

Date: 20 / 03 / 2018			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Assume GF(2). Calculate the following quantities:

a. $1 \oplus 1 = 0$

b. $1 \oplus 1 \oplus 1 = (1 \oplus 1) \oplus 1 = 0 \oplus 1 = 1$

c. $1 \cdot 0 = 0$

d. $1 \cdot 0 \cdot 1 = 0$

e. $[1 \ \ 1 \ \ 1] \oplus [1 \ \ 0 \ \ 1] = [1\oplus 1 \quad 1 \oplus 0 \quad 1 \oplus 1] = [0 \ \ 1 \ \ 0]$

f. $[0 \ \ 1 \ \ 1] \oplus [0 \ \ 1 \ \ 1] = [0 \oplus 0 \quad 1 \oplus 1 \quad 1 \oplus 1] = [0 \ \ 0 \ \ 0]$ Fact : $\underline{x} \oplus \underline{x} = \underline{0}$

g. $[1 \ \ 0 \ \ 1] \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = [(1\cdot 1)\oplus(0\cdot 0)\oplus(1\cdot 1) \quad (1\cdot 0)\oplus(0\cdot 1)\oplus(1 \cdot 0)] = [0 \ \ 0]$

Block matrix multiplication

Alternatively, $[1 \ \ 0 \ \ 1] \begin{bmatrix} \underline{r}^{(1)} \\ \underline{r}^{(2)} \\ \underline{r}^{(3)} \end{bmatrix} = \underline{r}^{(1)} \oplus \underline{r}^{(3)} = [1 \ \ 0] \oplus [1 \ \ 0] = [0 \ \ 0]$

2. A codeword $\underline{x}$ $[1 \ \ 1 \ \ 0 \ \ 1]$ is sent over the BSC. Suppose the error pattern is $\underline{e} = [0 \ \ 0 \ \ 1 \ \ 1]$. Find the observed vector at the receiver.

$\underline{y} = \underline{x} \oplus \underline{e} = [1 \ \ 1 \ \ 0 \ \ 1] \oplus [0 \ \ 0 \ \ 1 \ \ 1] = [1 \ \ 1 \ \ 1 \ \ 0]$.

Alternatively, $\underline{e} = [0 \ \ 0 \ \ 1 \ \ 1]$ means that the last two bits of $\underline{x}$ are received in error.

3. A codeword $\underline{x}$ $[1 \ \ 1 \ \ 0 \ \ 1]$ is sent over the BSC. Suppose the observed vector at the receiver is $\underline{y} = [0 \ \ 1 \ \ 1 \ \ 1]$. Find the error pattern.

$\underline{y} = \underline{x} \oplus \underline{e}$

$\underline{x} \oplus \underline{y} = \cancel{\underline{x} \oplus \underline{x}} + \underline{e}$

$\underline{e} = \underline{x} \oplus \underline{y}$

$$\begin{array}{r} [1 \ \ 1 \ \ 0 \ \ 1] \\ \oplus \ [0 \ \ 1 \ \ 1 \ \ 1] \\ \hline = [1 \ \ 0 \ \ 1 \ \ 0] \end{array}$$

So, these 1s correspond to the positions that $\underline{x}$ and $\underline{y}$ are different.

Alternatively, the error pattern indicates the locations of errors in the observed vector, $\underline{y}$.

4. A codeword is sent over the BSC.
Suppose the observed vector at the receiver is $\underline{y} = [0 \ \ 1 \ \ 1 \ \ 1]$ and the error pattern is $\underline{e} = [0 \ \ 0 \ \ 1 \ \ 1]$.
Find the transmitted codeword.

$\underline{y} = \underline{x} \oplus \underline{e}$

$\underline{y} \oplus \underline{e} = \underline{x} \oplus \cancel{\underline{e} \oplus \underline{e}}$

$$\underline{x} = \underline{y} \oplus \underline{e} = \begin{array}{r} [0 \ \ 1 \ \ 1 \ \ 1] \\ \oplus \ [0 \ \ 0 \ \ 1 \ \ 1] \\ \hline [0 \ \ 1 \ \ 0 \ \ 0] \end{array}$$

Alternatively, the error pattern says that the last two bits in the observed vector are received incorrectly. So, we need to flip their values to get the transmitted codeword.

ECS 452: In-Class Exercise # 12.1

Date: 30 / 03 / 2018			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups after the midterm.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. A linear block code uses the following generator matrix $\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$. $\rightarrow g^{(1)}$, $g^{(2)}$

   a. Find the code length n **The generator marix has 4 columns.** **n = 4**

   b. Find the codeword for the message $\mathbf{b} = [0 \quad 1]$

   $$\underline{x} = \underline{b}\,G = [0 \quad 1]\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} = [1 \quad 0 \quad 0 \quad 1]$$

   Direct multiplication like this is OK when we only have to find one codeword. However, in the next part, we need to find all of them; so we will use other methods.

   c. Find the codebook for this code.

   Method : $\underline{x} = \sum_i b_i g^{(i)}$ where b_i = the i^{th} element in $\underline{b}$ and $g^{(i)}$ = the i^{th} row of G

$\mathbf{b}$	$\mathbf{x}$
00	0000 ← $0 \cdot \underline{g}^{(1)} \oplus 0 \cdot \underline{g}^{(2)} = \underline{0}$
01	1001 ← $0 \cdot \underline{g}^{(1)} \oplus 1 \cdot \underline{g}^{(2)} = \underline{g}^{(2)}$
10	0111 ← $1 \cdot \underline{g}^{(1)} \oplus 0 \cdot \underline{g}^{(2)} = \underline{g}^{(1)}$
11	1110 ← $1 \cdot \underline{g}^{(1)} \oplus 1 \cdot \underline{g}^{(2)} = \underline{g}^{(1)} \oplus \underline{g}^{(2)}$

   Method 2: $\underline{b} = [b_1 \quad b_2]$

   $$\underline{x} = \underline{b}\,G = [b_2 \quad b_1 \quad b_1 \quad b_1 \oplus b_2]$$

   The first bit is simply b_2.

   The second and third bits are the same as b_1.

   The last bit is the sum of b_1 and b_2.

2. Each row of the table below correspond to a code that uses **single-parity-check**. The error pattern is given. Find the corresponding values of codeword length *n*, code dimension *k*, and Indicate whether the given error pattern is detectable.

   \# 1s is odd   \# 1s is even

Error Pattern	codeword length *n*	code dimension *k*	$\underline{e}$ is detectable (Yes or No?)
$\underline{e} = [1 \quad 0 \quad 0 \quad 1]$	4	3	No
$\underline{e} = [1 \quad 0 \quad 1 \quad 0 \quad 1]$	5	4	Yes
$\underline{e} = [0 \quad 0 \quad 0 \quad 0 \quad 1]$	5	4	Yes
$\underline{e} = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0]$	7	6	No

The length of vectors $\underline{e}, \underline{x}, \underline{y}$ should all be the same. $(\underline{y} = \underline{x} \oplus \underline{e})$ So, n = length of $\underline{e}$

For single-parity-check code, $k = n - 1$

ECS 452: In-Class Exercise # 12.2

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **30** / **03** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Consider a block code whose generator matrix is

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

1. Find the code length n

   The #columns of G is 6

2. Find the length k of each message block

   The #rows of G is 3

3. In the table below, list all possible data (message) vectors $\underline{\mathbf{b}}$ in the leftmost column (one in each row). Then, find the corresponding codewords $\underline{\mathbf{x}}$ and their weights in the second and third columns, respectively.

copy

$\underline{\mathbf{b}}$	$\underline{\mathbf{x}}$	$w(\underline{\mathbf{x}})$
000	000000	0
001	001110	3
010	010011	3
011	011101	4
100	100101	3
101	101011	4
110	110110	4
111	111000	3

$$\underline{x} = \underline{b}G = [b_1\ b_2\ b_3]\,G$$

$$= [\underbrace{b_1\ \ b_2\ \ b_3}_{\text{simply copy the bits from } \underline{b}}\ \ b_1 \oplus b_3\ \ b_2 \oplus b_3\ \ b_1 \oplus b_2]$$

The 4th column of G is $[1\ 0\ 1]^T$. Therefore, the 4th element of $\underline{x}$ is the sum of the 1st and 3rd elements of $\underline{b}$.

The 5th column of G is $[0\ 1\ 1]^T$. Therefore, the 5th element of $\underline{x}$ is the sum of the 2nd and 3rd elements of $\underline{b}$.

The 6th column of G is $[1\ 1\ 0]^T$. Therefore, the 6th element of $\underline{x}$ is the sum of the 1st and 2nd elements of $\underline{b}$.

4. Find the minimum distance $d_{\min}$ for this code.

min

3

ECS 452: In-Class Exercise #13

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Date: **03** / **04** / 2017			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. Consider a linear block code that uses parity checking on a square array:

b_1	b_2	$p_2 = b_1 \oplus b_2$
b_3	b_4	$p_3 = b_3 \oplus b_4$
$b_1 \oplus b_3 = p_1$	$p_4 = b_2 \oplus b_4$	

Each parity bit p_i is calculated such that the corresponding row or column has even parity.
Suppose the following bits arrangement is used in the codeword:

$$\underline{\mathbf{x}} = (b_1 \quad p_1 \quad p_2 \quad b_2 \quad b_3 \quad b_4 \quad p_3 \quad p_4).$$

a. Find the generator matrix $\mathbf{G}$.

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Alternatively, use the square array:

① put $\underline{b}$ inside the square

$$\begin{matrix} 0 & 1 & \xrightarrow{\Sigma} 1 \\ 1 & 0 & \xrightarrow{\Sigma} 1 \\ \Sigma\downarrow & \downarrow\Sigma & \\ 1 & 1 & \end{matrix}$$

② calculate the parity bits

③ put the values in the corresponding positions

0 1 1 1 1 0 1 1

b. Find the codeword for the message $\underline{\mathbf{b}} = [0 \quad 1 \quad 1 \quad 0]$.

$$\underline{x} = \underline{b}G = [0\ 1\ 1\ 0]\begin{bmatrix} g^{(1)} \\ g^{(2)} \\ g^{(3)} \\ g^{(4)} \end{bmatrix} = g^{(2)} \oplus g^{(3)} = \begin{matrix} 0\,0\,1\,1\,0\,0\,0\,1 \\ \oplus\ 0\,1\,0\,0\,1\,0\,1\,0 \\ \hline 0\,1\,1\,1\,1\,0\,1\,1 \end{matrix}$$

c. Find the parity-check matrix $\mathbf{H}$.

Identity matrix in the data positions becomes identity matrix in the parity positions

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} = \mathbf{H}$$

Bit values in the parity positions are transposed and put in the data positions

ECS 452: In-Class Exercise #14

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 10 / 04 / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Consider a block code whose generator matrix is

$$\mathbf{G} = \left(\begin{array}{ccc|ccc} \multicolumn{3}{c|}{I_3} & \multicolumn{3}{c}{P} \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right)$$

a. Find the parity check matrix **H** of this code.

$$H = \left[\begin{array}{ccc|ccc} \multicolumn{3}{c|}{P^T} & \multicolumn{3}{c}{I} \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

b. Suppose we receive $\underline{\mathbf{y}}$ = 111001.

i. Find the syndrome vector $\underline{\mathbf{s}}$.

$$\underline{s} = \underline{y} H^T = \left[\begin{pmatrix}1\\0\\1\end{pmatrix} + \begin{pmatrix}0\\1\\1\end{pmatrix} + \begin{pmatrix}1\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\1\end{pmatrix}\right]^T = \begin{pmatrix}0\\0\\1\end{pmatrix}^T = [0\ 0\ 1]$$

ii. Find the decoded codeword $\hat{\underline{\mathbf{x}}}$

The syndrome $\underline{s}$ is the same as the last column of H.

Therefore, $\hat{\underline{e}} = [0\ 0\ 0\ 0\ 0\ 1]$ and

$$\hat{\underline{x}} = \hat{\underline{y}} - \hat{\underline{e}} = \hat{\underline{y}} \oplus \hat{\underline{e}} = [\underbrace{1\,1\,1}\,0\,0\,0]$$

iii. Find the decoded message $\hat{\underline{\mathbf{b}}}$.

$$\hat{\underline{b}} = [1\ 1\ 1]$$

From G, we have I_3 in the front, so the message $\underline{b}$ will be the first three bits of the codeword $\underline{x}$.

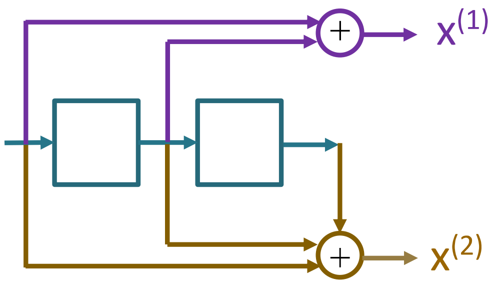
ECS 452: In-Class Exercise #15

Date: **20** / **04** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

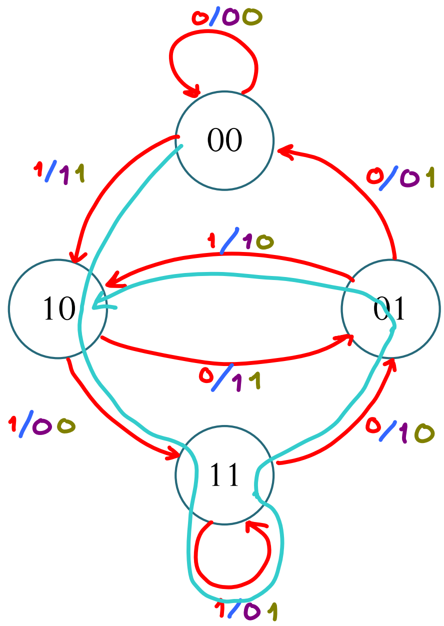
Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups after the midterm.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Consider a convolution encoder represented by the following diagram



(a) Draw the corresponding state (transition) diagram



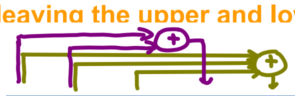
First, observe that this encoder uses a shift register with two FFs which is the same as the one discussed in lecture. Therefore, the arrows will be the same as what we had in the lecture.
Note, however, that the connections that produce the outputs are different from the encoder in lecture. Therefore, we simply need to find the outputs

b	s_1	s_2	$x^{(1)}$	$x^{(2)}$
0	0	0	0	0
1	0	0	1	1
0	0	1	0	1
1	0	1	1	0
0	1	0	1	1
1	1	0	0	0
0	1	1	1	0
1	1	1	0	1

(b) Suppose the information bits (the message bits) are $\underline{\mathbf{b}}$ = 11101.
Find the corresponding codeword $\underline{\mathbf{x}}$

i. by using the direct method (filling out the table below) without the help of the state diagram from part (a).

Note that the final output is one row vector resulting from interleaving the upper and lower outputs.



b	s_1	s_2	$x^{(1)}$	$x^{(2)}$
1	0	0	1	1
1	1	0	0	0
1	1	1	0	1
0	1	1	1	0
1	0	1	1	0

$\underline{\mathbf{x}}$ = 1 1 0 0 0 1 1 0 1 0

and

ii. by "tracing" the corresponding path on the state diagram derived an part (a)
Draw/highlight your trace on the state diagram in part (a) using different pen color.

See the trace in the diagram on the

$\underline{\mathbf{x}}$ = 1 1 0 0 0 1 1 0 1 0

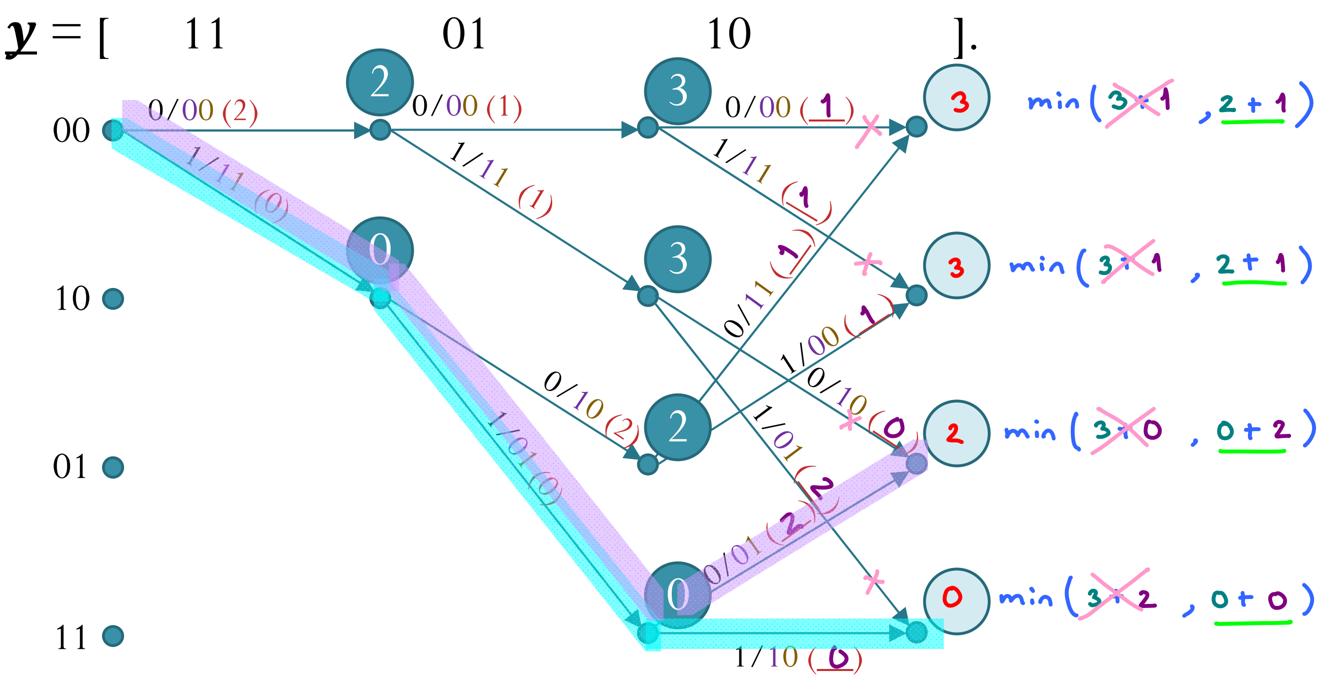
ECS 452: In-Class Exercise #16

Date: 24 / 04 / 2018			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Consider a convolutional encoder whose trellis diagram is given below



1. Suppose the data vector is $\underline{\mathbf{b}}$ = [110]. Find the corresponding codeword $\underline{\mathbf{x}}$.

Read from the highlighted path: [110101]

2. Suppose that we observe $\underline{\mathbf{y}}$ = 110110 at the input of the minimum distance decoder. The decoder uses Viterbi's algorithm.
   a. Write down
      (1) all the distance values on the branches and
      (2) the (chosen) cumulative distance values inside all the circles
      in the figure above.
   b. Find the decoded codeword $\hat{\underline{\mathbf{x}}}$ and the decoded message $\hat{\underline{\mathbf{b}}}$.

Read from the highlighted path:

$\hat{\underline{\mathbf{x}}}$ = **[110110]** $\hat{\underline{\mathbf{b}}}$ = **[111]** .

No error in the received vector

ECS 452: In-Class Exercise #17

Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups after the midterm.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 27 / 04 / 2018			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. Suppose $\vec{\mathbf{v}} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ and $\vec{\mathbf{u}} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$.

   a. Find $\langle \vec{\mathbf{v}}, \vec{\mathbf{u}} \rangle = (4)(2) + (0)(1) + (-3)(-2) = 8+0+6 = 14$

For $\vec{v} = (v_1, v_2, v_3)^T$ and $\vec{u} = (u_1, u_2, u_3)^T$, we have $\langle \vec{v}, \vec{u} \rangle = v_1u_1 + v_2u_2 + v_3u_3$

   b. Find $\langle \vec{\mathbf{u}}, \vec{\mathbf{u}} \rangle = 2^2 + 1^2 + (-2)^2 = 4+1+4 = 9$

For $\vec{u} = (u_1, u_2, u_3)^T$, we have $\langle \vec{u}, \vec{u} \rangle = u_1u_1 + u_2u_2 + u_3u_3 = u_1^2 + u_2^2 + u_3^2$

   c. Find $\langle \vec{\mathbf{v}}, \vec{\mathbf{v}} \rangle = 4^2 + 0^2 + (-3)^2 = 16+9 = 25$

Similar to part (b), for $\vec{v} = (v_1, v_2, v_3)^T$, we have $\langle \vec{v}, \vec{v} \rangle = v_1^2 + v_2^2 + v_3^2$

   d. Find $\|\vec{\mathbf{v}}\| \equiv \sqrt{\langle \vec{v}, \vec{v} \rangle} = \sqrt{25} = 5$

by definition

   e. Find $\|\vec{\mathbf{u}}\| \equiv \sqrt{\langle \vec{u}, \vec{u} \rangle} = \sqrt{9} = 3$

   f. Find $\mathrm{proj}_{\vec{\mathbf{u}}} \vec{\mathbf{v}} \equiv \frac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{u}, \vec{u} \rangle} \vec{u} = \frac{14}{9} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 28/9 \\ 14/9 \\ -28/9 \end{pmatrix} \approx \begin{pmatrix} 3.1111 \\ 1.5556 \\ -3.1111 \end{pmatrix}$

by definition

   g. Let $\vec{\mathbf{z}} = \vec{\mathbf{v}} - \mathrm{proj}_{\vec{\mathbf{u}}} \vec{\mathbf{v}}$. Find $\langle \vec{\mathbf{u}}, \vec{\mathbf{z}} \rangle$.

Direct calculation:

$\vec{z} = (4, 0, -3)^T - \frac{1}{9}(28, 14, -28)^T = \frac{1}{9}(8, -14, 1)^T$

$\langle \vec{u}, \vec{z} \rangle = \frac{1}{9}\left((2)(8) + (1)(-14) + (-2)(1)\right) = \frac{1}{9} \times 0 = 0.$

We know that $\underbrace{\vec{p} = \mathrm{proj}_{\vec{u}} \vec{v}}_{\vec{u} \text{ is parallel to } \vec{p}}$ and $\underbrace{\vec{o} = \vec{v} - \vec{p}}_{\vec{z} \text{ is exactly the same as } \vec{o}}$ are always orthogonal.

Therefore, $\vec{u}$ and $\vec{z}$ are orthogonal and hence $\langle \vec{u}, \vec{z} \rangle = 0$.

Alternatively,

$\langle \vec{u}, \vec{z} \rangle = \langle \vec{u}, \vec{v} - \mathrm{proj}_{\vec{u}} \vec{v} \rangle = \langle \vec{u}, \vec{v} \rangle - \langle \vec{u}, \frac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{u}, \vec{u} \rangle} \vec{u} \rangle = \langle \vec{u}, \vec{v} \rangle - \frac{\cancel{\langle \vec{u}, \vec{u} \rangle}}{\cancel{\langle \vec{u}, \vec{u} \rangle}} \langle \vec{v}, \vec{u} \rangle = 0$

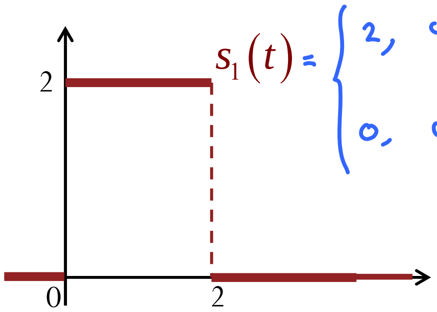
ECS 452: In-Class Exercise #**18**

Date: **01** / **05** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

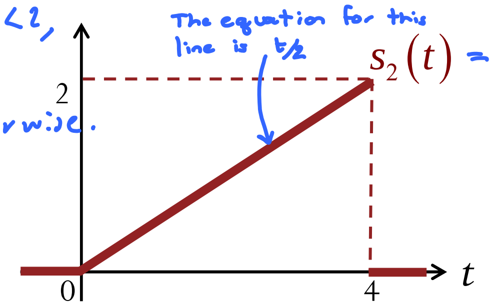
Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Consider the two signals $s_1(t)$ and $s_2(t)$ shown below.



$$s_1(t) = \begin{cases} 2, & 0<t<2, \\ 0, & \text{otherwise.} \end{cases}$$



The equation for this line is $t/2$

$$s_2(t) = \begin{cases} \frac{1}{2}t, & 0<t<4, \\ 0, & \text{otherwise} \end{cases}$$

a. Find the energy of each signal.

$$E_1 = \int_{-\infty}^{\infty} s_1^2(t)dt = \int_0^2 2^2 dt = \int_0^2 4\,dt = 4t\Big|_0^2 = 4\times 2 = 8$$

$$E_2 = \int_{-\infty}^{\infty} s_2^2(t)dt = \int_0^4 \left(\frac{t}{2}\right)^2 dt = \int_0^4 \frac{t^2}{4}dt = \frac{t^3}{12}\Big|_0^4 = \frac{4\times4\times4}{12} = \frac{16}{3}$$

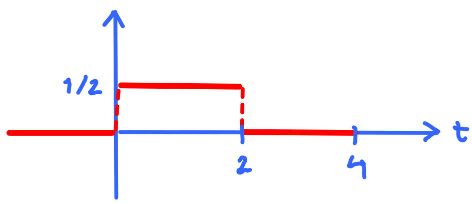
b. Find their inner product $\langle s_1(t), s_2(t)\rangle$.

note that this is 2 (not 4) because $s_1(t)=0$ when $2<t<4$.

$$\langle s_1(t), s_2(t)\rangle = \int_{-\infty}^{\infty} s_1(t)s_2(t)dt = \int_0^2 2\times\frac{t}{2}\,dt = \int_0^2 t\,dt = \frac{t^2}{2} = \frac{4}{2} = 2$$

c. Find and plot $\text{proj}_{s_1(t)} s_2(t)$.

$$\text{proj}_{s_1} s_2 = \frac{\langle s_2, s_1\rangle}{\langle s_1, s_1\rangle} s_1 = \frac{2}{8} = \frac{1}{4}s_1 = \begin{cases} 1/2, & 0<t<2, \\ 0, & \text{otherwise.} \end{cases}$$



Continue on the next page

ECS 452: In-Class ~~Exercise # _~~ Additional Example

Instructions

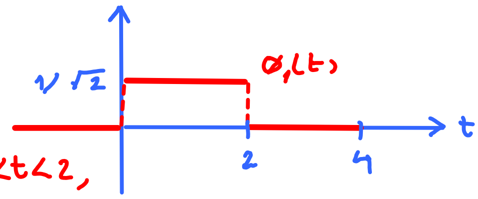
1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: _ _ / _ _ / 2018			
Name	ID (last 3 digits)		

Continue from Exercise #18.

d. Suppose the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) is used to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used as axes to represent $s_1(t)$ and $s_2(t)$.

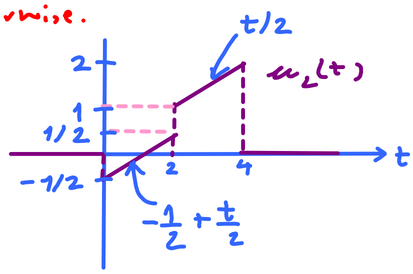
i. Find and plot $\phi_1(t)$



$u_1(t) = s_1(t)$ $\quad$ $s_1 = 2\sqrt{2}\,\phi_1 \approx 2.83\phi_1$

$$\phi_1(t) = \frac{u_1(t)}{\sqrt{E_{u_1}}} = \frac{s_1(t)}{\sqrt{E_1}} = \frac{1}{2\sqrt{2}} s_1 = \begin{cases} 1/\sqrt{2}, & 0<t<2, \\ 0, & \text{otherwise.} \end{cases}$$

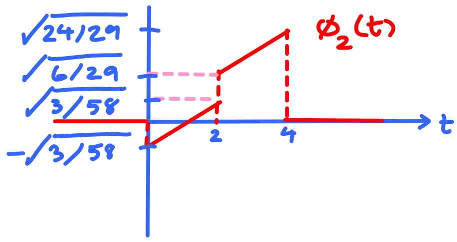
ii. Find and plot $\phi_2(t)$.



$u_2(t) = s_2(t) - \text{proj}_{u_1} s_2 = s_2(t) - \frac{1}{4}s_1(t)$

$$E_{u_2} = \int_{-\infty}^{\infty} u_2^2(t)\,dt = \int_0^2 \left(-\frac{1}{2}+\frac{t}{2}\right)^2 dt + \int_2^4 \left(\frac{t}{2}\right)^2 dt$$

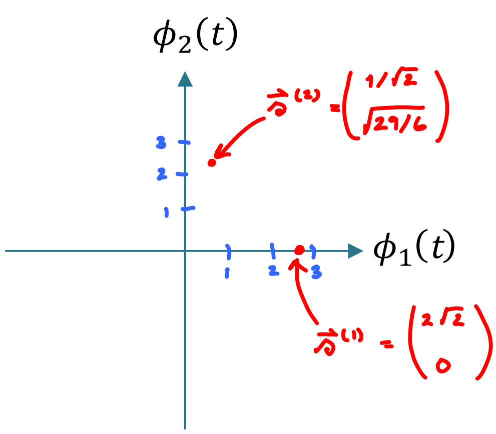
$$= \frac{1}{6} + \frac{14}{3} = \frac{1+28}{6} = \frac{29}{6}$$



$$\phi_2(t) = \frac{u_2(t)}{\sqrt{E_{u_2}}} = \sqrt{\frac{6}{29}}\,u_2(t) = \begin{cases} \sqrt{\frac{3}{58}}(t-1), & 0<t<2, \\ \sqrt{\frac{3}{58}}\,t, & 2\le t<4, \\ 0, & \text{otherwise.} \end{cases}$$

$$s_2(t) = \frac{1}{4}s_1(t) + u_2(t) = \frac{1}{4}2\sqrt{2}\,\phi_1(t) + \sqrt{\frac{29}{6}}\,\phi_2(t) = \frac{1}{\sqrt{2}}\phi_1(t) + \sqrt{\frac{29}{6}}\,\phi_2(t) \approx 0.71\,\phi_1(t) + 2.2\,\phi_2(t)$$

iii. Find the two vectors, $\vec{\mathbf{s}}^{(1)}$ and $\vec{\mathbf{s}}^{(2)}$, that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new axes based on $\phi_1(t)$ and $\phi_2(t)$. Draw the corresponding constellation in the figure below.



$\langle \vec{s}^{(1)}, \vec{s}^{(1)} \rangle$

$$E_{s_1} = \int_{-\infty}^{\infty} s_1^2(t)\,dt = \|s_1(t)\|^2 = \|\vec{s}^{(1)}\|^2 = (2\sqrt{2})^2 + 0^2 = 8$$

$$E_{s_2} = (1/\sqrt{2})^2 + (\sqrt{29/6})^2 = \frac{16}{3}$$

$$\langle s_1(t), s_2(t)\rangle = \langle \vec{s}^{(1)}, \vec{s}^{(2)} \rangle = \left(2\sqrt{2}\times\frac{1}{\sqrt{2}}\right) + \left(0\times\sqrt{\frac{29}{6}}\right)$$

$$= 2$$

ECS 452: In-Class Exercise # 19

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 11 / 05 / 2018			
Name	ID (last 3 digits)		
Prapun	5	5	5

A digital communication system transmits a stream of bits by mapping each block of three bits to one of the possible waveforms $s_1(t), s_2(t), \ldots, s_M(t)$. The waveform is then transmitted via a communication channel which corrupts the waveform by independently adding a white noise process $N(t)$ whose power spectral density is given by $S_N(f) \equiv 16$ across all frequency.

$\frac{N_0}{2}$

a. What is the value of *M*?

Three bits $\Rightarrow$ 8 possibilities for the block

$\Rightarrow$ need 8 different waveforms to represent 8 distinct block patterns

b. Suppose we apply GSOP to the *M* waveforms and get two orthonormal axes $\phi_1(t)$ and $\phi_2(t)$. Let $N_j = \langle N(t), \phi_j(t) \rangle$. Find

See 7.26f

i. $\mathbb{E}[N_1] = 0$

ii. $\mathrm{Var}[N_1] = \frac{N_0}{2} = 16$

iii. $\sigma_{N_1} = \sqrt{\mathrm{Var}[N_1]} = \sqrt{16} = 4$

iv. $\mathbb{E}[N_1 N_2] = 0$

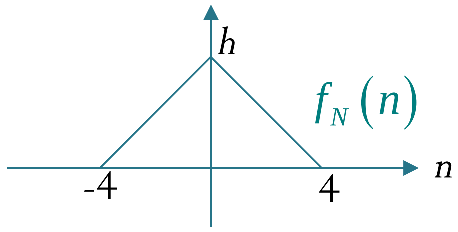
ECS 452: In-Class Exercise #**20**

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **15** / **05** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

In a binary antipodal signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{\overset{s^{(1)}}{-2}, \overset{s^{(2)}}{2}\}$ with $p_1 = P[S=-2] = 0.6$ and $p_2 = P[S=2] = 0.4$. The message is corrupted by an independent additive noise N whose pdf is shown below:

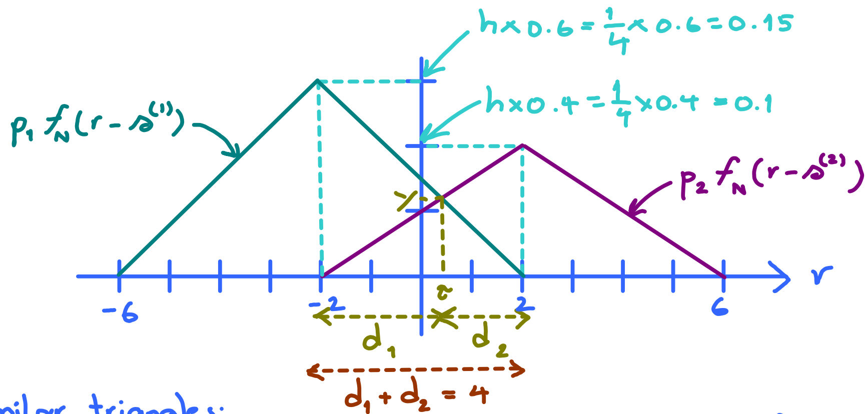


a. What is the value of h?

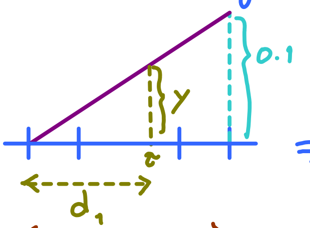
To be a pdf, we need $\int_{-\infty}^{\infty} f_N(n)\,dn = 1$.

$\frac{1}{2} \times 8 \times h = 1 \Rightarrow h = \frac{1}{4}$

b. Suppose the received symbol is $R = r$. Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

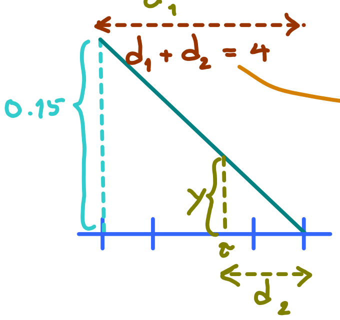


Use similar triangles:



$\Rightarrow \frac{y}{d_1} = \frac{0.1}{4}$

$y = \frac{d_1 \times 0.1}{4} = \frac{d_2 \times 0.15}{4}$



$\Rightarrow \frac{y}{d_2} = \frac{0.15}{4}$

$d_1 = \frac{3}{2} d_2 \Rightarrow \frac{3}{2} d_2 + d_2 = 4$

$d_2 = 4 \times \frac{2}{5} = \frac{8}{5}$

$\tau = 2 - d_2 = 2 - \frac{8}{5} = \frac{2}{5} = 0.4$

$$\hat{s}_{\text{MAP}}(r) = \begin{cases} -2, & -6 < r < 0.4, \\ 2, & 0.4 < r < 6, \\ \text{any}, & \text{otherwise} \end{cases} = \begin{cases} -2, & r < 0.4, \\ 2, & r \geq 0.4. \end{cases}$$