Name ________________________ ID3 _ _ _

ECS 452: Digital Communication Systems **2017/2**

HW 9 — Due: Not Due

Lecturer: Prapun Suksompong, Ph.D.

Problem 1. In a binary antipodal signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-3, 3\}$ with $p_1 = P[S = -3] = 0.3$ and $p_2 = P[S = 3] = 0.7$. The message is corrupted by an independent additive noise N which is uniform on $[-4, 4]$.

(a) Find the pdf of the noise.

(b) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(c) Evaluate the error probability of the MAP detector.

Problem 2. In a binary antipodal signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-3, 3\}$ with $p_1 = P[S = -3] = 0.3$ and $p_2 = P[S = 3] = 0.7$. The message is corrupted by an independent additive exponential noise N whose pdf is

$$f_N(n) = \begin{cases} \frac{1}{2}e^{-n/2}, & n \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(b) Evaluate the error probability of the MAP detector.

Problem 3. In a ternary signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-1, 1, 4\}$ with $p_1 = P[S = -1] = 0.3 = p_2 = P[S = 1]$ and $p_3 = P[S = 4] = 0.4$. The message is corrupted by an independent additive Gaussian noise $N \sim \mathcal{N}(0, 2)$.

(a) Find the average signal energy[1] E_s.

(b) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

[1] Same as "average symbol energy" or "average energy per symbol" or "average energy per signal"

(c) Indicate the decision regions of the MAP detector in part (b).

(d) Evaluate the error probability of the MAP detector.

Problem 4. In a **standard** quaternary signaling scheme, the message S is equiprobably selected from the alphabet set $\mathcal{S} = \left\{-\frac{3d}{2}, -\frac{d}{2}, \frac{d}{2}, \frac{3d}{2}\right\}$. The message is corrupted by an independent additive exponential noise N whose pdf is

$$f_N(n) = \begin{cases} \lambda e^{-\lambda n}, & n \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the average symbol energy.

(b) Find the average energy per bit.

(c) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(d) Evaluate the error probability of the MAP detector.

(e) Let $\lambda = \frac{1}{\sigma}$. (This is to set $\operatorname{Var} N = \sigma^2$ as in the case for Gaussian noise.) Plot $\frac{E_b}{\sigma^2}$ vs. probability of error $P(\mathcal{E})$. Consider $\frac{E_b}{\sigma^2}$ from -30 to 10 dB.