Name ________________________ ID3 _ _ _

ECS 452: Digital Communication Systems **2017/2**

HW 8 — Due: May 18, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 8 pages.

(b) (1 pt) Work and write your answers **directly on these provided sheets** (not on other blank sheet(s) of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 8.1. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

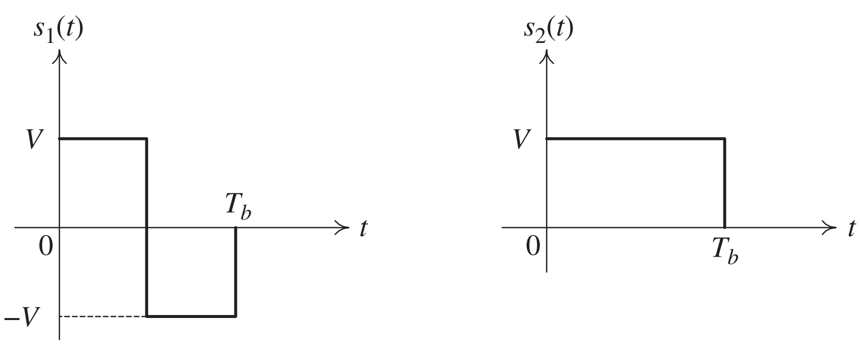

Figure 8.1: Signal set for Problem 1

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

(c) Find the two vectors that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part. Draw the corresponding constellation.

Problem 2. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 8.2. Note that V, α and T_b are some positive constants.

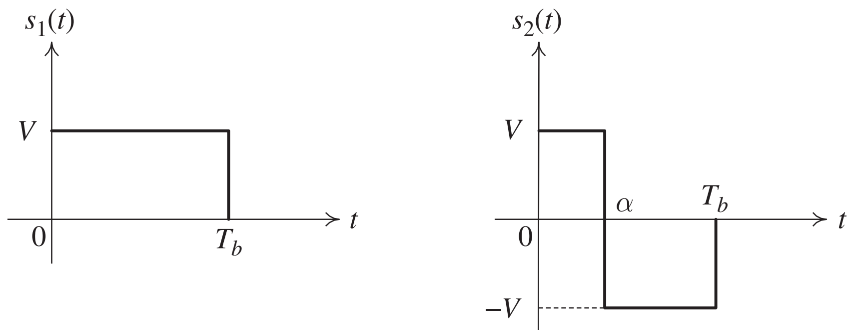

Figure 8.2: Signal set for Problem 2

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

(c) Plot $\phi_1(t)$ and $\phi_2(t)$ when $\alpha = \frac{T_b}{4}$.

(d) Find the two vectors $\mathbf{s}^{(1)}$ and $\mathbf{s}^{(2)}$ that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part.

(e) Draw the corresponding constellation when $\alpha = \frac{T_b}{4}$.

(f) Draw $\mathbf{s}^{(2)}$ when $\alpha = \frac{k}{10}T_b$ where $k = 1, 2, \ldots, 9$.

Problem 3. In a ternary signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-1, 1, 4\}$ with $p_1 = P[S = -1] = 0.41$, $p_2 = P[S = 1] = 0.08$ and $p_3 = P[S = 4] = 0.51$. Find the average signal energy E_s.

Problem 4. Consider a ternary constellation. Assume that the three vectors are equiprobable.

(a) Suppose the three vectors are

$$\mathbf{s}^{(1)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{s}^{(2)} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \text{ and } \mathbf{s}^{(3)} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Find the corresponding average energy per symbol.

(b) Suppose we can shift the above constellation to other location; that is, suppose that the three vectors in the constellation are

$$\mathbf{s}^{(1)} = \begin{pmatrix} 0 - a_1 \\ 0 - a_2 \end{pmatrix}, \mathbf{s}^{(2)} = \begin{pmatrix} 3 - a_1 \\ 0 - a_2 \end{pmatrix}, \text{ and } \mathbf{s}^{(3)} = \begin{pmatrix} 3 - a_1 \\ 3 - a_2 \end{pmatrix}.$$

Find a_1 and a_2 such that corresponding average energy per symbol is minimum.

Problem 5. Let $g(t)$ be band-limited to 100 Hz and $E_g = 8$. Suppose the GSOP (where the waveforms are applied in the order given) is used to find the orthonormal basis for the following signal sets. Draw the corresponding constellation for each signal set. (Hint: We can solve this problem without actually applying GSOP.)

(a) $\{s_m(t) = (2m-5)\, g(t) \cos(2,000\pi t), \quad m = 1, 2, 3, 4\}$

(b) $\left\{s_m(t) = g(t) \cos\left(2,000\pi t + \frac{\pi}{2}(m-1)\right), \quad m = 1, 2, 3, 4\right\}$

(c) $\left\{s_m(t) = 1_{[0,2]}(t) \cos(\pi m t), \quad m = 1, 2, 3\right\}$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 6. Suppose $s_1(t) = \operatorname{sinc}(5t)$ and $s_2(t) = \operatorname{sinc}(7t)$. Note that in this class, we define $\operatorname{sinc}(x) = \frac{\sin x}{x}$. Find

(a) E_{s_1},

(b) E_{s_2}, and

(c) $\langle s_1(t), s_2(t) \rangle$.

Hint: Use Parseval's theorem to evaluate the above quantities in the frequency domain. (Review: See 2.43 on p. 22 and Example 2.44 on p. 23 of ECS332 2017 lecture notes and Problem 6 in ECS332 2017 HW3.)

Problem 7. Prove the following facts with the help of Fourier transform.
(Hint: inner product in the frequency domain, Parseval's theorem)

(a) The energy of $p(t) = g(t)\cos(2\pi f_c t + \phi)$ is $E_g/2$.

(b) $g(t)\cos(2\pi f_c t)$ and $-g(t)\sin(2\pi f_c t)$ are orthogonal.

Is there any condition(s) on $g(t)$ that we need to assume?