ECS 452: Digital Communication Systems	**2017/2**

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HW 2 — Due: Feb 23, 4 PM

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Instructions

(a) This assignment has 6 pages.

(b) (1 pt) Work and write your answers **directly on these provided sheets** (not on other blank sheet(s) of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Continue from the Example 2.40 in the lecture. A memoryless source emits two possible message Y(es) and N(o) with probability 0.9 and 0.1, respectively.

(a) Calculate the entropy (per source symbol) of this source.

(b) Find the expected codeword length per symbol of the Huffman binary code for the third-order extensions of this source.

Problem 2. Consider a BSC whose crossover probability for each bit is $p = 0.35$. Suppose $P[X = 0] = 0.45$.

(a) Draw the channel diagram.

(b) Find the channel matrix $\mathbf{Q}$.

(c) Find the joint pmf matrix $\mathbf{P}$.

(d) Find the row vector $\underline{\mathbf{q}}$ which contains the pmf of the channel output Y.

(e) Analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

$\hat{x}(y)$	$\hat{x}(0)$	$\hat{x}(1)$	$P(\mathcal{C})$	$P(\mathcal{E})$
y				
$1 - y$				
1				
0				

Problem 3. Consider a BAC whose $Q(1|0) = 0.35$ and $Q(0|1) = 0.55$. Suppose $P[X = 0] = 0.4$.

(a) Draw the channel diagram.

(b) Find the joint pmf matrix $\mathbf{P}$.

(c) Find the row vector $\underline{\mathbf{q}}$ which contains the pmf of the channel output Y.

(d) Analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

$\hat{x}(y)$	$\hat{x}(0)$	$\hat{x}(1)$	$P(\mathcal{C})$	$P(\mathcal{E})$
y				
$1-y$				
1				
0				

Problem 4. Consider a DMC whose $\mathcal{X} = \{1, 2, 3\}$, $\mathcal{Y} = \{1, 2, 3\}$, and $\mathbf{Q} = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$.

Suppose the input probability vector is $\underline{\mathbf{p}} = [0.2, 0.4, 0.4]$.

(a) Find the joint pmf matrix $\mathbf{P}$.

(b) Find the row vector $\underline{\mathbf{q}}$ which contains the pmf of the channel output Y.

(c) Find the following probabilities:

(i) $P[X = 1]$

(ii) $P[Y = 2]$

(iii) $P[X = 1, Y = 2]$

(iv) $P[Y = 2 | X = 1]$

(v) $P[X = 1 | Y = 2]$

(vi) Find the error probability of the naive decoder.

(vii) Find the error probability of the (DIY) decoder $\hat{x}(y) = 4 - y$.

Problem 5. A DMC has $\mathcal{X} = \{0, 1\}$ and $\mathcal{Y} = \{1, 2, 3\}$. The following decoding table is used to decode the channel output.

y	$\hat{x}(y)$
1	0
2	1
3	0

Suppose the channel output string is 212213221133122122132.

(a) Find the corresponding decoded string.

(b) Suppose the channel input string is produced from an ASCII source encoder by the command `dec2bin(SourceString,7)` in MATLAB. Assume that there is no channel decoding error. Find the corresponding source string.
(There is no need to actually use MATLAB to solve this problem.
Hint: See pages 14 and 23-24 of the slides for Chapter 2.)

Extra Questions

Here are some optional questions for those who want more practice.

Problem 6. *Optimal code lengths that require one bit above entropy*: The source coding theorem says that the Huffman code for a random variable X has an expected length strictly less than $H(X) + 1$. Give an example of a random variable for which the expected length of the Huffman code (without any source extension) is very close to $H(X) + 1$.

Problem 7. Continue from the Example 2.40 in the lecture and Problem 1. A memoryless source emits two possible message Y(es) and N(o) with probability 0.9 and 0.1, respectively.

(a) Use `MATLAB` to find the expected codeword length **per (source) symbol** of the Huffman binary code for the fourth-order extensions of this source.

(b) Use `MATLAB` to plot the expected codeword length **per (source) symbol** of the Huffman binary code for the nth-order extensions of this source for $n = 1, 2, \ldots, 8$. Attach the **printout** of your plot.