

# ECS 452: In-Class Exercise # 8

## Instructions

- Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
- Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
- Do not panic.**

Date: <b>23 / 02 / 2018</b>			
Name			ID (last 3 digits)
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1. Consider two random variables  $X$  and  $Y$  whose joint pmf matrix is given by

$$\mathbf{P} = \begin{array}{c|ccccc}
 X \backslash Y & 1 & 2 & 3 & 4 & 5 \\
 \hline
 1 & 1/8 & 0 & 0 & 1/8 & 0 \\
 2 & 1/8 & 1/8 & 1/8 & 0 & 1/8 \\
 3 & 0 & 1/8 & 0 & 1/8 & 0 \\
 \hline
 & \downarrow \Sigma & \downarrow \Sigma & \downarrow \Sigma & \downarrow \Sigma & \downarrow \Sigma \\
 & 1/4 & 1/4 & 1/8 & 1/4 & 1/8
 \end{array}
 \begin{array}{l}
 p(x) \\
 \Sigma \rightarrow 1/4 \\
 \Sigma \rightarrow 1/2 \\
 \Sigma \rightarrow 1/4
 \end{array}$$

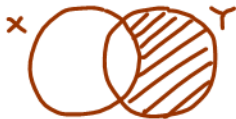
Calculate the following quantities.

a.  $H(X, Y) = 8 \times \left(-\frac{1}{8} \log_2 \frac{1}{8}\right) = -\log_2 2^{-3} = 3$   
 ↑  
 There are this many " $\frac{1}{8}$ " in the P matrix.

b.  $H(X) = \left(-\frac{1}{2} \log_2 \frac{1}{2}\right) + 2 \left(-\frac{1}{4} \log_2 \frac{1}{4}\right)$   
 $= \left(-\frac{1}{2} \log_2 2^{-1}\right) + \left(-\frac{1}{2} \log_2 2^{-2}\right) = \frac{1}{2} + 1 = \frac{3}{2}$

c.  $H(Y) = 3 \times \left(-\frac{1}{4} \log_2 \frac{1}{4}\right) + 2 \times \left(-\frac{1}{8} \log_2 \frac{1}{8}\right)$   
 $= \left(-\frac{3}{4} (-2)\right) + \left(-\frac{1}{4} (-3)\right) = \frac{6}{4} + \frac{3}{4} = \frac{9}{4}$

d.  $H(Y|X) = H(X, Y) - H(X) = 3 - \frac{3}{2} = \frac{3}{2}$



e. Q matrix

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$Q(y|x) = P[Y=y|X=x] = \frac{P[X=x, Y=y]}{P[X=x]} = \frac{p(x,y)}{p(x)}$$

so, to get the Q matrix from the P matrix,

we scale each row of the Q matrix by the corresponding  $\frac{1}{p(x)}$

$$\begin{array}{c}
 \left[ \begin{array}{ccccc}
 1/8 & 0 & 0 & 1/8 & 0 \\
 1/8 & 1/8 & 1/8 & 0 & 1/8 \\
 0 & 1/8 & 0 & 1/8 & 0
 \end{array} \right]
 \begin{array}{l}
 \xrightarrow{\times 4} \\
 \xrightarrow{\times 2} \\
 \xrightarrow{\times 4}
 \end{array}
 \begin{array}{c|ccccc}
 X \backslash Y & 1 & 2 & 3 & 4 & 5 \\
 \hline
 1 & 1/2 & 0 & 0 & 1/2 & 0 \\
 2 & 1/4 & 1/4 & 1/4 & 0 & 1/4 \\
 3 & 0 & 1/2 & 0 & 1/2 & 0
 \end{array}
 \end{array}$$

With the Q matrix, we can now find  $H(Y|X)$  using another method:

$$\begin{array}{l}
 H(Y|X=1) = \log_2 2 = 1 \\
 H(Y|X=2) = \log_2 4 = 2 \\
 H(Y|X=3) = \log_2 2 = 1
 \end{array}
 \Rightarrow H(Y|X) = \sum_x p(x) H(Y|X=x)$$

$$= \frac{1}{4} \times 1 + \frac{1}{2} \times 2 + \frac{1}{4} \times 1 = \frac{3}{2}$$

element in the P matrix

channel input probability