ECS 452: In-Class Exercise # 8

Date: 23 / 02 / 2018	
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Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Consider two random variables X and Y whose joint pmf matrix is given by

$$\mathbf{P} = \begin{array}{c} x\backslash y \\ 1 \\ 2 \\ 3 \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccccc} 1/8 & 0 & 0 & 1/8 & 0 \\ 1/8 & 1/8 & 1/8 & 0 & 1/8 \\ 0 & 1/8 & 0 & 1/8 & 0 \end{array}\right] \\ \begin{array}{ccccc} \downarrow\Sigma & \downarrow\Sigma & \downarrow\Sigma & \downarrow\Sigma & \downarrow\Sigma \\ 1/4 & 1/4 & 1/8 & 1/4 & 1/8 \end{array} \end{array} \begin{array}{c} p(x) \\ \xrightarrow{\Sigma} \frac{1}{4} \\ \xrightarrow{\Sigma} \frac{1}{2} \\ \xrightarrow{\Sigma} \frac{1}{4} \\ \\ \end{array}$$

Calculate the following quantities.

a. $H(X,Y) = 8 \times \left(-\frac{1}{8}\log_2 \frac{1}{8}\right) = -\log_2 2^{-3} = 3$

There are this many "$\frac{1}{8}$" in the P matrix.

b. $H(X) = \left(-\frac{1}{2}\log_2 \frac{1}{2}\right) + 2\left(-\frac{1}{4}\log_2 \frac{1}{4}\right)$

$= \left(-\frac{1}{2}\log_2 2^{-1}\right) + \left(-\frac{1}{2}\log_2 2^{-2}\right) = \frac{1}{2} + 1 = \frac{3}{2}$

c. $H(Y) = 3 \times \left(-\frac{1}{4}\log_2 \frac{1}{4}\right) + 2 \times \left(-\frac{1}{8}\log_2 \frac{1}{8}\right)$

$= \left(-\frac{3}{4}(-2)\right) + \left(-\frac{1}{4}(-3)\right) = \frac{6}{4} + \frac{3}{4} = \frac{9}{4}$

d. $H(Y|X) = H(X,Y) - H(X) = 3 - \frac{3}{2} = \frac{3}{2}$

With the Q matrix, we can now find $H(Y|X)$ using another method:

$H(Y|X=1) = \log_2 2 = 1$
$H(Y|X=2) = \log_2 4 = 2$
$H(Y|X=3) = \log_2 2 = 1$

$\Rightarrow H(Y|X) = \sum_x p(x) H(Y|X=x) = \frac{1}{4}\times 1 + \frac{1}{2}\times 2 + \frac{1}{4}\times 1 = \frac{3}{2}$

e. **Q** matrix

$P(A|B) = \frac{P(A\cap B)}{P(B)}$

$Q(y|x) = P[Y=y|X=x] = \frac{P[X=x, Y=y]}{P[X=x]} = \frac{p(x,y)}{p(x)}$

element in the P matrix

channel input probability

So, to get the Q matrix from the P matrix, we scale each row of the Q matrix by the corresponding $\frac{1}{p(x)}$

$$\left[\begin{array}{ccccc} 1/8 & 0 & 0 & 1/8 & 0 \\ 1/8 & 1/8 & 1/8 & 0 & 1/8 \\ 0 & 1/8 & 0 & 1/8 & 0 \end{array}\right] \begin{array}{c} \xrightarrow{\times 4} \\ \xrightarrow{\times 2} \\ \xrightarrow{\times 4} \end{array} \begin{array}{c} x\backslash y \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccccc} 1/2 & 0 & 0 & 1/2 & 0 \\ 1/4 & 1/4 & 1/4 & 0 & 1/4 \\ 0 & 1/2 & 0 & 1/2 & 0 \end{array}\right] \end{array}$$