ECS 452: In-Class Exercise # 6

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 13 / 02 / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. Consider a DMC whose transition matrix $\mathbf{Q}$ is

$x \backslash y$	1	2	3	4
1	0.10	0.20	0.30	0.40
2	0.25	0.25	0.25	0.25
3	0.50	0.30	0.10	0.10

$\times 0.2$, $\times 0.4$, $\times 0.4$

$x \backslash y$	1	2	3	4
1	0.02	0.04	0.06	0.08
2	0.10	0.10	0.10	0.10
3	0.20	0.12	0.04	0.04

$= P$

Σ Σ Σ Σ

0.32 0.26 0.20 0.22

Suppose the input probability vector is $\underline{\mathbf{p}} = [0.2 \quad 0.4 \quad 0.4]$.

a) Find the joint pmf matrix $\mathbf{P}$. Put your answer next to the $\mathbf{Q}$ matrix above.

b) Find the output probability vector $\underline{\mathbf{q}}$.

[0.32 0.26 0.20 0.22]

c) Suppose the naïve decoder is used. Find the corresponding $P(\mathcal{E})$.

$\hat{x}(y)$: 1 2 3 4

$x \backslash y$	1	2	3	4
1	0.02	0.04	0.06	0.08
2	0.10	0.10	0.10	0.10
3	0.20	0.12	0.04	0.04

$\mathbf{P} =$

$P(C) = 0.02 + 0.10 + 0.04 = 0.16$

$P(\mathcal{E}) = 1 - P(C) = 1 - 0.16 = 0.84$

d) Suppose the following decoder is used. Find the corresponding $P(\mathcal{E})$.

y	$\hat{x}(y)$
1	1
2	1
3	2
4	3

$\hat{x}(y)$: 1 1 2 3

$x \backslash y$	1	2	3	4
1	0.02	0.04	0.06	0.08
2	0.10	0.10	0.10	0.10
3	0.20	0.12	0.04	0.04

$\mathbf{P} =$

$P(C) = 0.02 + 0.04 + 0.10 + 0.04 = 0.2$

$P(\mathcal{E}) = 1 - P(C) = 1 - 0.2 = 0.8$

e) Suppose the decoder is $\hat{x}(y) = 2.5 - |y - 2.5|$

Find the corresponding $P(\mathcal{E})$.

y	$y - 2.5$	$2.5 - \lvert y - 2.5 \rvert$
1	-1.5	1
2	-0.5	2
3	0.5	2
4	1.5	1

$\hat{x}(y)$: 1 2 2 1

$x \backslash y$	1	2	3	4
1	0.02	0.04	0.06	0.08
2	0.10	0.10	0.10	0.10
3	0.20	0.12	0.04	0.04

$\mathbf{P} =$

$P(C) = 0.02 + 0.10 + 0.10 + 0.08 = 0.3$

$P(\mathcal{E}) = 1 - P(C) = 1 - 0.3 = 0.7$