

ECS 452: In-Class Exercise # 5

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **09 / 02 / 2018**

Name	ID <small>(last 3 digits)</small>		
Prapun	5	5	5

1. No need to provide any explanation for this question.

Consider a DMC whose samples of input and output are provided below

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

x: 1 1 1 0 1 0 1 0 1 1 1 1 1 0 0
 y: 1 1 1 0 1 0 1 1 1 1 1 0 1 1 0

Estimate the following quantities:

a. $\mathcal{X} = S_x = \{0, 1\}$

b. $P[X=0] \approx \frac{5}{15} = \frac{1}{3}$

c. $p(1) \approx P[X=1] = \frac{15-5}{15} = \frac{10}{15} = \frac{2}{3}$

d. $p_Y(0) \approx P[Y=0] = \frac{4}{15}$

e. $\underline{p} \approx [\frac{1}{3} \quad \frac{2}{3}]$

f. $q(1) \approx P[Y=1] = \frac{15-4}{15} = \frac{11}{15}$

g. $P[Y=0 | X=0] \approx \frac{3}{5}$

h. $p_{Y|X}(1|0) = P[Y=1 | X=0]$
 $\approx 1 - \frac{3}{5} = \frac{2}{5}$

i. $Q(0|1) = P[Y=0 | X=1]$
 $\approx \frac{1}{10}$

j. $Q(1|1) = P[Y=1 | X=1]$
 $\approx 1 - \frac{1}{10} = \frac{9}{10}$

k. Matrix \underline{Q}
 $\approx \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 3/5 & 2/5 \\ 1/10 & 9/10 \end{bmatrix} \end{matrix}$

l. $P[X=0, Y=0]$
 $= P[Y=0 | X=0] P[X=0]$
 $\approx \frac{3}{5} \times \frac{1}{3} = \frac{1}{5}$

Alternatively, among the 15 samples, $(x, y) = (0, 0)$ happens 3 times. Therefore,

$P[X=0, Y=0] \approx \frac{3}{15} = \frac{1}{5}$