

ECS 452: In-Class Exercise # 2

Instructions

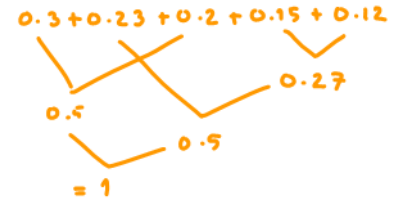
1. Separate into groups of no more than three persons. Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 16 / 01 / 2018			
Name			ID (last 3 digits)
Prapun			5 5 5

1. Consider a DMS whose source alphabet is {a,e,c,n,t}. The probabilities for these five symbols are shown in the table below:

x	a	e	c	n	t
$p(x)$	0.30	0.23	0.20	0.15	0.12

First, we check that this is a pmf:



Consider two codes (for source coding) below.
 The left column is for Code A. The right column is for Code B.
 The first row defines these codes via their codebooks.

<p>Codebook for Code A</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>x</td> <td>a</td> <td>e</td> <td>c</td> <td>n</td> <td>t</td> </tr> <tr> <td>$c(x)$</td> <td>10</td> <td>01</td> <td>11</td> <td>000</td> <td>100</td> </tr> </table> <p style="color: purple;">$l(x)$ 2 2 2 3 3</p>	x	a	e	c	n	t	$c(x)$	10	01	11	000	100	<p>Codebook for Code B</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>x</td> <td>a</td> <td>e</td> <td>c</td> <td>n</td> <td>t</td> </tr> <tr> <td>$c(x)$</td> <td>1</td> <td>00</td> <td>010</td> <td>0111</td> <td>01101</td> </tr> </table> <p style="color: green;">$l(x)$ 1 2 3 4 5</p>	x	a	e	c	n	t	$c(x)$	1	00	010	0111	01101
x	a	e	c	n	t																				
$c(x)$	10	01	11	000	100																				
x	a	e	c	n	t																				
$c(x)$	1	00	010	0111	01101																				
<p>Is Code A prefix-free?</p> <p style="color: red;">No. The codeword "10" for "a" is a prefix of the codeword "100" for "t".</p>	<p>Is Code B prefix-free?</p> <p style="color: blue;">Yes. No codeword is a prefix of another codeword.</p>																								
<p>Suppose the DMS above is encoded by Code A. Find the expected codeword length.</p> <p style="color: blue;"> $E[l(x)] = \sum_x l(x) p(x)$ $= 2 \times 0.3 + 2 \times 0.23 + 2 \times 0.2 + 3 \times 0.15 + 3 \times 0.12$ $= 2(0.3 + 0.23 + 0.2) + 3(0.15 + 0.12)$ $= 2(0.73) + 3(0.27)$ $= 1.46 + 0.81 = 2.27 \text{ bits}$ </p>	<p>Suppose the DMS above is encoded by Code B. Find the expected codeword length.</p> <p style="color: blue;"> $E[l(x)] = \sum_x l(x) p(x)$ $= 1 \times 0.3 + 2 \times 0.23 + 3 \times 0.20 + 4 \times 0.15 + 5 \times 0.12$ $= \underbrace{0.3 + 0.46}_{0.76} + \underbrace{0.6 + 0.6 + 0.6}_{1.8}$ $= 2.56 \text{ bits}$ </p>																								

Draw the code tree for code B.

