ECS 452: In-Class Exercise # 2

Instructions

1. Separate into groups of no more than three persons. Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 16 / 01 / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. Consider a DMS whose source alphabet is {a,e,c,n,t}. The probabilities for these five symbols are shown in the table below:

x	a	e	c	n	t
$p(x)$	0.30	0.23	0.20	0.15	0.12

First, we check that this is a pmf:

$0.3+0.23+0.2+0.15+0.12$

0.27

0.5

0.5

= 1

Consider two codes (for source coding) below.
The left column is for Code A. The right column is for Code B.
The first row defines these codes via their codebooks.

Codebook for Code A

x	a	e	c	n	t
$c(x)$	10	01	11	000	100
$\ell(x)$	2	2	2	3	3

Is Code A prefix-free?

No. The codeword "10" for "a" is a prefix of the codeword "100" for "t".

Suppose the DMS above is encoded by Code A.
Find the expected codeword length.

$$E[\ell(X)] = \sum_x \ell(x)\,p(x)$$

$$= 2\times0.3 + 2\times0.23 + 2\times0.2 + 3\times0.15 + 3\times0.12$$

$$= 2(0.3+0.23+0.2) + 3(0.15+0.12)$$

$$= 2(0.73) + 3(0.27)$$

$$= 1.46 + 0.81 = 2.27 \text{ bits}$$

Codebook for Code B

x	a	e	c	n	t
$c(x)$	1	00	010	0111	01101
$\ell(x)$	1	2	3	4	5

Is Code B prefix-free?

Yes. No codeword is a prefix of another codeword.

Suppose the DMS above is encoded by Code B.
Find the expected codeword length.

$$E[\ell(X)] = \sum_x \ell(x)\,p(x)$$

$$= 1\times0.3 + 2\times0.23 + 3\times0.20 + 4\times0.15 + 5\times0.12$$

$$= \underbrace{0.3+0.46}_{0.76} + \underbrace{0.6+0.6+0.6}_{1.8}$$

$$= 2.56 \text{ bits}$$

Draw the code tree for code B.